MATH 250
FALL 2021 NAME:
EXAM 2

1. [10 pts.] Solve the homogeneous equation:

$$y' = -\frac{x^2 + y^2}{2xy}.$$

2. [10 pts.] Solve the equation using a suitable substitution:

$$(2x + y + 1)\frac{dy}{dx} = 1.$$

3. [15 pts.] A thermometer is taken from inside a Taco Hell kitchen to the great outdoors where the air temperature is $-5°$ C. After 1 minute the thermometer reads $10°$ C, and after 4 minutes it reads $0°$ C. What is the temperature in the kitchen?

4. [20 pts.] A tank with a capacity of 500 liters originally contains 200 L of water with 30 kg of salt in solution. Water containing 0.3 kg of salt per liter begins entering the tank at a rate of 4 L/min, and the mixture is allowed to flow out of the tank at a rate of 2 L/min. Find the amount of salt in the tank at any time t before the tank is full. Also find the concentration, in kg/L, of salt in the tank at the instant when the tank is full.

5. [10 pts.] Given that $y = x$ is a solution to

$$(1 - x^2)y'' - 2xy' + 2y = 0$$

on interval $(-1, 1)$, use reduction of order to find a second linearly independent solution y_2.

6. [15 pts.] Find the particular solution to the IVP

$$6y'' - 11y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = -2.$$

7. [10 pts.] Find the general solution to

$$3y''' + 5y'' + y' - y = 0.$$

Some Integration Formulas:

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c,$$

$$\int \tan x\,dx = \ln|\sec x| + c, \quad \int \cot x\,dx = \ln|\sin x| + c, \quad \int \sec x\,dx = \ln|\sec x + \tan x| + c,$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + c.$$