MATH 250
FALL 2021
EXAM 1

NAME:

1. [10 pts.] Verify that the piecewise-defined function

$$y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

is a solution to $xy' - 2y = 0$ with interval of validity $(-\infty, \infty)$. (Take care finding y' when $x = 0$.)

2. [10 pts.] Find all points (x_0, y_0) on the xy-plane where the IVP

$$(3x^2 - 4y)y' = \sqrt{y}, \quad y(x_0) = y_0,$$

must have a unique solution.

3. [15 pts.] Solve the initial-value problem

$$\frac{dy}{dx} = \frac{1 - 2x}{y}, \quad y(1) = -2.$$

Put the solution in explicit form and find the largest interval of validity I for the solution.

4. [15 pts.] Solve the differential equation

$$\frac{dy}{dx} = x^2(1 + y)$$

to get the most inclusive one-parameter family of explicit solutions possible. What is the largest interval of validity for the solutions?

5. [15 pts.] Solve the linear IVP

$$\frac{dy}{dx} = \frac{y}{x} + 2x + 1, \quad y(-1) = 8.$$

Write the solution in explicit form with all absolute values eliminated. What is the largest interval of validity for the solution?

6. [15 pts.] Solve the exact equation

$$ye^{xy} - \frac{1}{y} + \left(xe^{xy} + \frac{x}{y^2}\right)y' = 0.$$

Some Integration Formulas:

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c,$$

$$\int \tan x\,dx = \ln|\sec x| + c, \quad \int \cot x\,dx = \ln|\sin x| + c, \quad \int \sec x\,dx = \ln|\sec x + \tan x| + c,$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + c.$$