MATH 250
FALL 2020
EXAM 4

NAME:

1. [30 pts.] Find two (linearly independent) series solutions to the differential equation

$$(x^2+1)y'' - 6y = 0$$

about the ordinary point $x = 0$.

2. [10 pts.] Use the definition of the Laplace transform to find $\mathcal{L}[f(t)]$ given that

$$f(t) = \begin{cases} t, & 0 \le t < 2 \\ 3-t, & t \ge 2 \end{cases}$$

3. [10 pts. each] Find the inverse Laplace transform.

(a) $\mathcal{L}^{-1}\left[\dfrac{15s}{2s^2+50}\right]$

(b) $\mathcal{L}^{-1}\left[\dfrac{s+1}{s^2-4s}\right]$

4. [30 pts.] Use the Method of Laplace Transforms to solve the initial-value problem

$$y'' + 9y = e^t, \quad y(0) = 0, \quad y'(0) = 0.$$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$u(t-a),\ a\geq 0$	$\dfrac{e^{-as}}{s}$	$s>0$
$\delta(t-a),\ a\geq 0$	e^{-as}	$s>0$
$(f*g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s>0$

$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$

$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$

$\mathcal{L}[t^n f(t)](s) = (-1)^n F^{(n)}(s)$

$\mathcal{L}[f(t-a)u(t-a)](s) = e^{-as}\mathcal{L}[f(t)](s)$

$\mathcal{L}[g(t)u(t-a)](s) = e^{-as}\mathcal{L}[g(t+a)](s)$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\sin x\cos y = \frac{1}{2}[\sin(x+y)+\sin(x-y)]$

$\cos x\cos y = \frac{1}{2}[\cos(x+y)+\cos(x-y)]$

$\sin x\sin y = \frac{1}{2}[\cos(x-y)-\cos(x+y)]$