MATH 250
FALL 2020
EXAM 2

NAME:

1. [15 pts.] Suppose that the temperature of a cup of tea obeys Newton's Law of Cooling. If the tea has a temperature of 200°F when first poured, and 1 minute later has cooled to 190°F in a room at 72°F, find when the tea reaches a temperature of 120°F.

2. [15 pts.] A tank is used to conduct certain hydrodynamic experiments. After one experiment the tank contains 200 liters of a dye solution having a concentration of 1 g/liter. To prepare for the next experiment, the tank is rinsed with fresh water flowing in at a rate of 2 liters/min, the well-stirred solution flowing out at the same rate. Find the time that must elapse for the tank to have a concentration of dye that is 1% of its original value.

3. [15 pts.] Using either the Wronskian or the definition of linear independence, show that the set of solutions $\{x, x^{-2}, x^{-2}\ln x\}$ to the differential equation

$$x^3y''' + 6x^2y'' + 4xy' - 4y = 0$$

is a linearly independent set of functions on the interval $(0, \infty)$. Form the general solution to the differential equation.

4. [10 pts.] Given that $y_1 = x + 1$ is a solution to

$$(1 - 2x - x^2)y'' + 2(1 + x)y' - 2y = 0,$$

use reduction of order to find a second solution

$$y_2 = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{y_1^2(x)}\,dx.$$

5. [10 pts. each] Find the general solution to each.

   (a) $2y'' - 7y' + 3y = 0$

   (b) $2y''' - 5y'' + 8y' - 20y = 0$

6. [15 pts.] Solve the initial-value problem:

$$y'' - 2y' + y = 0, \quad y(0) = 5, \quad y'(0) = 10.$$