MATH 250
FALL 2020
EXAM 1

NAME:

Absolute values in solutions to differential equations should be eliminated when it's reasonably straightforward to do so. Also solutions should be in explicit form when possible, unless doing so conflicts with the elimination of absolute values.

1. [10 pts.] $y = c_1e^{3x} + c_2e^{-x} - 2x$ is a two-parameter family of solutions to the differential equation $y'' - 2y' - 3y = 6x + 4$. Find a solution to the initial-value problem

$$y'' - 2y' - 3y = 6x + 4, \quad y(0) = 1, \quad y'(0) = -3.$$

2. [10 pts.] Solve the separable equation

$$\frac{dy}{dx} = \frac{x^2}{y(1 + x^3)}.$$

3. [10 pts.] Solve the initial-value problem

$$(2y - \sin y)y' + x = \sin x, \quad y(0) = 0.$$

4. [10 pts.] Solve the linear equation

$$2xy' - 3y = 9x^3.$$

5. [10 pts.] Solve the exact equation

$$x + \tan^{-1} y + \left(\frac{x + y}{1 + y^2}\right)\frac{dy}{dx} = 0.$$

6. [10 pts.] Solve the homogeneous equation

$$\frac{dy}{dx} = -\frac{4x + 3y}{2x + y}.$$

A few integration formulas:

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c,$$

$$\int \tan x\,dx = \ln|\sec x| + c, \quad \int \cot x\,dx = \ln|\sin x| + c, \quad \int \sec x\,dx = \ln|\sec x + \tan x| + c,$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + c.$$