Show all work (and answers) on the blank paper provided. Write nothing on this paper other than your name. **Name:**

1a	10	
1b	10	
2a	10	
2b	10	
3	25	
4	25	
5	25	
6	25	
EC	30	
total	140	
curve		
%		

1) Find the Laplace transform of each function.

    a. $\left(3 - e^{-2t}\right)^2$

    b. $\sin 3t \cos 3t$ (use the identity $\sin \alpha \cos \beta = \frac{1}{2}\sin(\alpha - \beta) + \frac{1}{2}\sin(\alpha + \beta)$)

2) Find the inverse Laplace function of each function.

    c. $\dfrac{s - 1}{2s^2 + s + 6}$

    a. $\dfrac{s - 10}{(s + 1)(s - 6)}$

3) Solve the initial value problem using the Method of Laplace Transforms:
$$y'' - 4y' + 5y = 4e^{3t}, \quad y(0) = 2, \quad y'(0) = 7$$

4) Solve the initial value problem using the Method of Laplace Transforms:
$$y'' + 5y' + 6y = tu(t - 2), \quad y(0) = 0, \quad y'(0) = 1$$

5) Solve the initial value problem using the Method of Laplace Transforms:
$$y'' + 2y' + 2y = \delta(t - \pi), \quad y(0) = 1, \quad y'(0) = 1$$

6) A mixing tank initially holds 800 L of a brine solution with a salt concentration of 0.3 kg/L. For the first 20 minutes of operation, Valve A is open, adding 16 L/min of brine containing a 0.6 kg/L salt concentration. After 20 minutes, Valve A is closed and Valve B is opened, adding a 0.2 kg/L concentration at 16 L/min. The exit valve C removes 16 L/min, thereby keeping the volume constant. Find the concentration of salt in the tank as a function of time.

Extra Credit
Two large tanks, each holding 100 L of liquid, are interconnected by pipes, with the liquid flowing from Tank A to Tank B at a rate of 3 L/min and from B to A at a rate of 1 L/min. The liquid inside each tank is kept well stirred. A brine solution with a concentration of 0.2 kg/L of salt flows into Tank A at a rate of 6 L/min. The (diluted) solution flows out of the system from Tank A at 4 L/min and from Tank B at 2 L/min. If, initially, Tank A contains pure water and Tank B contains 20 kg of salt, determine the mass of salt in each tank over time.