MATH 250
FALL 2008 NAME:
EXAM 1

1. [5 pts.] Write a differential equation that fits the physical description: The rate of change of the mass A of salt at time t is proportional to the square of the mass of salt present at time t.

2. [8 pts.] Let

$$\sqrt{1-y}\,\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=0.$$

Is the equation an ordinary or partial differential equation? Is the equation linear or nonlinear? What is the order of the equation? Identify the independent variable and dependent variable.

3. [5 pts. each] Determine whether the Existence-Uniqueness Theorem implies that the initial value problem has a unique solution.

   (a) $3\frac{dx}{dt}+4t=0,\ x(2)=\pi.$

   (b) $y\frac{dy}{dx}=-5x,\ y(1)=0.$

4. [10 pts.] Determine for which values of m the function $\varphi(x)=e^{mx}$ is a solution to $6\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0.$

5. [10 pts. each] Solve each initial value problem.

   (a) $\frac{dy}{d\theta}=y\sin\theta,\ y(\pi)=-3$

   (b) $\frac{dy}{dx}=x^2(1+y),\ y(0)=3$

   (c) $t^3\frac{dx}{dt}+3t^2x=t,\ x(2)=0$

6. [15 pts.] A pot of boiling water at 100 °C is removed from a stove at time $t=0$ and left to cool. After 6 minutes the water temperature has decreased to 85 °C, and another 6 minutes later it has dropped to 72 °C. Assuming Newton's Law of Cooling,

$$\frac{dT}{dt}=k(M-T),$$

determine the temperature of the kitchen. (Recall that M is ambient temperature and T is the temperature of the object.)