

1. 5 pts. each Classify each as an ordinary or partial differential equation, linear or nonlinear differential equation (if applicable), give the equation's order, and identify the independent and dependent variables.

(a) $\frac{d^2y}{dx^2} - 2x\frac{d^3y}{dx^3} + 2y^2 = 0.$

(b) $\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r} + kN, \quad k \text{ constant.}$

2. 10 pts. Determine whether $y - \ln y = x^2 + 1$ is an implicit solution to $\frac{dy}{dx} = \frac{2xy}{y-1}.$

3. 10 pts. Determine for which values of m the function $\varphi(x) = x^m$ is a solution to the equation $x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = 0.$

4. 10 pts. Solve $x\frac{dr}{dx} = \frac{1-5r^2}{7r}.$

5. 10 pts. Solve the initial value problem $\frac{dy}{dx} = 2\sqrt{y+1}\sin x, \quad y(\pi/2) = 0.$

6. 10 pts. Solve $y' = \frac{y}{x} + 2x + 1.$

7. 10 pts. Solve the initial value problem $t^3\frac{dx}{dt} + 3t^2x = t, \quad x(2) = 0.$

8. A differential equation has the direction field below.

- (a) 3 pts. Sketch the solution curve with initial condition $y(-2) = -2.$
- (b) 3 pts. Sketch the solution curve with initial condition $y(0) = 0.$
- (c) 4 pts. What can be said about the behavior of the above solutions as $x \rightarrow \infty$ and $x \rightarrow -\infty$?

