MATH 242
SUMMER 2023 NAME:
EXAM 3

1. [10 pts.] Use the Lagrange multiplier method to find the absolute maximum and minimum values, if any, of $f(x,y) = x^2 + y^2$ subject to the constraint $2x^2 + 3xy + 2y^2 = 7$.

2. [10 pts.] Evaluate the double integral

$$\int_0^{\pi/2} \int_0^{\cos y} e^{\sin y} dx\, dy.$$

3. [10 pts.] Evaluate the double integral over the region R, choosing a convenient order of integration:

$$\iint_R x \sec^2(xy)\, dA, \quad R = \{(x,y) : 0 \le x \le \tfrac{\pi}{3},\, 0 \le y \le 1\}.$$

4. [10 pts.] Evaluate the double integral

$$\iint_R \frac{2y}{\sqrt{x^4+1}}\, dA,$$

where R is the region bounded by $x = 1$, $x = 2$, $y = x^{3/2}$, and $y = 0$.

5. [10 pts.] Evaluate using polar coordinates:

$$\int_{-4}^{4} \int_0^{\sqrt{16-y^2}} (16 - x^2 - y^2)\, dx\, dy$$

6. [10 pts.] Use a double integral and polar coordinates to find the volume of the solid bounded below by the paraboloid $z = x^2 + y^2 - x - y$ and above by the plane $z = 4 - x - y$.

7. [10 pts.] Find the volume of the region in space bounded by the graphs of $z = 9 - x^2$, $y = -x + 2$, $y = 0$, and $z = 0$, with $x \ge 0$.

8. [10 pts.] Rewrite the integral

$$\int_0^1 \int_{-2}^{2} \int_0^{\sqrt{4-y^2}} dz\, dy\, dx$$

in the order $dy\, dz\, dx$, then evaluate the resulting integral.

9. [10 pts.] Use cylindrical coordinates to find the volume of the region that is inside both the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$.

10. 10 pts. Evaluate

$$\iiint_D (x^2 + y^2)\, dV,$$

where D is the region outside the sphere $x^2 + y^2 + z^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 16$.