MATH 242
SPRING 2023 NAME:
EXAM 2

1. [10 pts.] Find the domain and range of $F(x, y) = \ln(3 + \cos(x + y))$.

2. [10 pts.] Sketch three level curves for $f(x, y) = x^3 - y$, labeling each curve with its z value.

3. [10 pts.] Use the Two-Path Test to prove that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4 + y^4}$$

4. [10 pts.] Evaluate the limit

$$\lim_{(x,y)\to(1,2)} \frac{\sqrt{y} - \sqrt{x+1}}{y - x - 1}.$$

5. [10 pts.] Find the partial derivatives indicated.

   (a) Find φ_x and φ_y for $\varphi(x, y) = y^3 \tan^{-1}(x^2y)$.

   (b) Find ψ_z and ψ_{xy} for $\psi(x, y, z) = \dfrac{y}{x + z}$.

6. Let

$$f(x, y) = \begin{cases} \dfrac{2xy^2}{x^2 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

   (a) [10 pts.] Is f continuous at $(0, 0)$? If not, prove it.

   (b) [5 pts.] Is f differentiable at $(0, 0)$? If not, why not?

   (c) [10 pts.] Evaluate $f_x(0, 0)$ and $f_y(0, 0)$, if they exist.

7. [10 pts.] Given $w = \cos(2x)\sin(3y)$ with $x = t/2$ and $y = t^4$, use an appropriate chain rule to find dw/dt. Express the answer in terms of t.

8. Let $f(x, y) = 12 - 4x^2 - y^2$.

   (a) [5 pts.] Find the gradient of f.

   (b) [5 pts.] Find the unit vector that gives the direction of maximum increase (i.e. steepest ascent) for f at $(1, 2)$.

   (c) [5 pts.] Find the unit vectors that give the directions of zero change for f at $(1, 2)$.

   (d) [10 pts.] Let C be the path of steepest descent on the surface $z = f(x, y)$ beginning at $(1, 2, 4)$, and let C_0 be the projection of C onto the xy-plane. Find an equation for C_0.

9. [10 pts.] Compute the directional derivative of

$$f(x,y) = 13e^{xy}$$

at the point $(1,0)$ in the direction $\langle 5, 12 \rangle$.

10. Consider the surface S given by $f(x,y) = (x+y)e^{xy}$.

    (a) [10 pts.] Find an equation of the tangent plane to S at the point $(2,0,2)$.

    (b) [5 pts.] Use the tangent plane to estimate the value of $f(1.95, 0.05)$.

11. [10 pts.] Find the critical points of

$$f(x,y) = 10 - x^3 - y^3 - 3x^2 + 3y^2,$$

then determine whether each critical point corresponds to a local maximum, local minimum, or saddle point.