MATH 242
SUMMER 2023
EXAM 1

NAME:

1. $\boxed{\text{5 pts.}}$ Find the vector with length 12 pointing in the direction opposite that of $\langle 4, -9\rangle$.

2. $\boxed{\text{10 pts.}}$ Find the velocity $\mathbf{v}$ of an ocean freighter that is traveling 60° north of east at 35 km/hr.

3. $\boxed{\text{10 pts.}}$ Give a geometric description of the set of points (x, y, z) satisfying

$$x^2 - 6x + y^2 + z^2 - 20z + 9 > 0.$$

4. $\boxed{\text{10 pts.}}$ Find the values of x and y such that the points $(1, 2, 3)$, $(4, 7, 1)$ and $(x, y, 9)$ are collinear.

5. $\boxed{\text{10 pts. each}}$ Let $\mathbf{u} = \langle 2, 0, -1\rangle$ and $\mathbf{v} = \langle -3, 8, 4\rangle$.
   (a) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ to the nearest tenth of a degree.
   (b) Find $\text{proj}_{\mathbf{v}}\,\mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.

6. $\boxed{\text{10 pts.}}$ For what value of c is the vector $\mathbf{u} = \langle 4, -3, c\rangle$ orthogonal to $\mathbf{v} = \langle 2, 9, -1\rangle$?

7. $\boxed{\text{10 pts.}}$ Find a vector orthogonal to both $\langle 0, -1, 2\rangle$ and $\langle 8, -2, -1\rangle$.

8. $\boxed{\text{10 pts.}}$ Find a parametrization for the line through $(-3, -3, 8)$ that is perpendicular to both the y-axis and $\mathbf{u} = \langle 0, 3, -5\rangle$.

9. $\boxed{\text{10 pts.}}$ Determine whether the lines are parallel, intersecting, or skew:

$$\mathbf{r}(t) = \langle 5 + 2t, 3 + 3t, 1 - t\rangle, \quad \mathbf{R}(s) = \langle 13 - 3s, 13 - 4s, 4 - 2s\rangle.$$

   If they intersect, find the point(s) of intersection.

10. $\boxed{\text{10 pts.}}$ Given that $\mathbf{r}'(t) = \langle \cos t, 1 - 2e^{-t}, 1 - 2e^{t}\rangle$ and $\mathbf{r}(0) = \langle 1, 1, 1\rangle$, find $\mathbf{r}(t)$.

11. $\boxed{\text{10 pts.}}$ Find an equation of the plane containing the points $(1, 1, 0)$, $(3, -1, 4)$ and $(1, 2, 3)$.

12. $\boxed{\text{10 pts.}}$ Find an equation of the line where the planes $x + 2y - 3z = 1$ and $x + y + z = 2$ intersect.

13. $\boxed{\text{10 pts.}}$ Find the domain of $\mathbf{r}(t) = \sqrt{4 - t^2}\,\mathbf{i} + \sqrt{t}\,\mathbf{j} - \dfrac{5}{\sqrt{1 + t}}\,\mathbf{k}$.

14. $\boxed{\text{10 pts.}}$ Find the arc length of the curve given by $\mathbf{r}(t) = \langle 2t^{9/2}, t^3\rangle$ for $0 \le t \le 3$.

15. $\boxed{\text{10 pts. each}}$ Let C be the curve given by $\mathbf{r}(t) = \langle t, \ln t \rangle$ for $t > 0$.

(a) Find the unit tangent vector $\mathbf{T}$ for $\mathbf{r}$.

(b) Find the curvature κ for $\mathbf{r}$.

(c) Find the point on C where the curvature attains a maximum. What is the maximum curvature value?