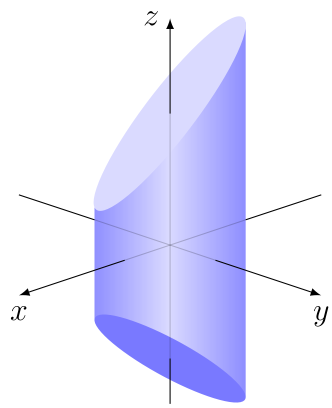
Math 242
Summer 2013
Exam 3

Name:

1. [10 pts.] Find the volume of the wedge of the cylinder $x^2 + 4y^2 = 4$ created by the planes $z = 3 - x$ and $z = x - 3$.



2. [10 pts.] Evaluate the triple integral

$$\int_1^6 \int_0^{4-2y/3} \int_0^{12-2y-3z} \frac{1}{y} dx\, dz\, dy.$$

3. [10 pts.] Evaluate the integral

$$\int_0^4 \int_0^{\sqrt{2}/2} \int_x^{\sqrt{1-x^2}} e^{-x^2-y^2} dy\, dx\, dz$$

in cylindrical coordinates.

4. [10 pts.] Evaluate the integral

$$\iiint_D (x^2 + y^2 + z^2)^{5/2} dV$$

in spherical coordinates, where D is the unit ball.

5. [10 pts.] Evaluate the line integral

$$\int_C (y - z),$$

where C is the helix $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, t \rangle$ for $0 \le t \le 2\pi$.

6. [10 pts.] Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle -y, x \rangle$, and the curve C is the semicircle $\mathbf{r}(t) = \langle 4\cos t, 4\sin t \rangle$ for $0 \le t \le \pi$.

7. [10 pts.] Show that the vector field $\mathbf{F}(x, y, z) = \langle 2xy^3z^4, 3x^2y^2z^4, 4x^2y^3z^3 \rangle$ is conservative on $\mathbb{R}^3$, and then determine a potential function.

8. 10 pts. Evaluate

$$\int_C \nabla(e^{-x} \cos y) \cdot d\mathbf{r},$$

where C is the line segment from $(0, 0)$ to $(\ln 2, 2\pi)$.

9. 10 pts. Use the circulation form of Green's Theorem to evaluate the line integral $\oint_{\partial R} \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle 2xy, x^2 - y^2 \rangle$ and R is the region bounded by $y = x(2 - x)$ and $y = 0$.

10. 10 pts. Find the divergence of $\mathbf{F}(x, y, z) = \langle -2y, 3x, z \rangle$.

11. 10 pts. Find the curl of $\mathbf{F}(x, y, z) = \langle x^2 - y^2, xy, z \rangle$.

12. Consider the half-cylinder consisting of the set of points

$$\{(r, \theta, z) : r = 4,\ 0 \leq \theta \leq \pi,\ 0 \leq z \leq 7\}$$

in polar coordinates.

(a) 5 pts. Give a parametric description of the half-cylinder in the form

$$\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle.$$

(b) 10 pts. Find the area of the half-cylinder by evaluating the appropriate surface integral.

13. 10 pts. Let $\mathbf{F}(x, y, z) = \langle 2y, -z, x \rangle$, and let C be the circle $x^2 + y^2 = 12$ in the plane $z = 0$. Evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ by evaluating the surface integral in Stokes' Theorem using an appropriate choice of surface Σ.