MATH 242
SUMMER 2013
EXAM 2

NAME:

1. $\boxed{\text{10 pts.}}$ Determine at what points in $\mathbb{R}^2$ the function $h(x,y) = \ln(x^2 - 3y)$ is continuous.

2. $\boxed{\text{10 pts.}}$ Graph two level curves of the function $z = \sqrt{x^2 + 4y^2}$, labeling each curve with its z-value.

3. $\boxed{\text{10 pts.}}$ Evaluate the limit

$$\lim_{(x,y)\to(2,1)} \frac{x^2 - 4y^2}{x - 2y}.$$

4. $\boxed{\text{10 pts.}}$ Use the Two-Path Test to prove that the limit

$$\lim_{(x,y)\to(0,0)} \frac{xy + y^3}{x^2 + y^2}$$

does not exist.

5. $\boxed{\text{10 pts. each}}$ Find the partial derivatives indicated.
   (a) Given $g(x,y) = x\ln(x^2 + y^2)$, find g_x and g_y.
   (b) Given $h(x,y,z) = \cos(x + 2y + 3z)$, find h_z and h_{zy}.

6. Let

$$f(x,y) = \begin{cases} -\dfrac{xy}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

   (a) $\boxed{\text{10 pts.}}$ Is f continuous at $(0,0)$? If not, prove it.
   (b) $\boxed{\text{5 pts.}}$ Is f differentiable at $(0,0)$? If not, why not?
   (c) $\boxed{\text{10 pts.}}$ If possible, evaluate $f_y(0,0)$.

7. $\boxed{\text{10 pts.}}$ Given $w = \cos(2x)\sin(3y)$ with $x = t/2$ and $y = t^4$, use an appropriate chain rule to find $w'(t)$. Express the answer in terms of t.

8. $\boxed{\text{10 pts.}}$ Use a chain rule to find z_s and z_t, where $z = xy - 2x + 3y$ with $x = \sin(s)$ and $y = \tan(t)$.

9. Let $f(x,y) = 2y - 3x^3$.
   (a) $\boxed{\text{5 pts.}}$ Find the gradient of f.
   (b) $\boxed{\text{5 pts.}}$ Find the unit vectors that give the direction of steepest ascent and steepest descent at $(1,2)$.
   (c) $\boxed{\text{10 pts.}}$ Let C be the path of steepest descent on the surface $z = f(x,y)$ beginning at $(1,2,1)$, and let C_0 be the projection of C onto the xy-plane. Find an equation for C_0.

10. $\boxed{\text{10 pts.}}$ Compute the directional derivative of

$$f(x, y) = e^x \sin y$$

at the point $(0, \pi/4)$ in the direction $\langle 1, \sqrt{3} \rangle$.

11. Consider the surface S given by $f(x, y) = (x + y)/(x - y)$.

    (a) $\boxed{\text{10 pts.}}$ Find an equation of the tangent plane to S at the point $(3, 2, 5)$.

    (b) $\boxed{\text{5 pts.}}$ Use the tangent plane to estimate the value of $f(2.95, 2.05)$.

12. $\boxed{\text{15 pts.}}$ Find the critical points of $f(x, y) = xye^{-x-y}$, then determine whether each critical point corresponds to a local maximum, local minimum, or saddle point.

13. $\boxed{\text{15 pts.}}$ Find the global extrema of the function $f(x, y) = 6 - x^2 - 4y^2$ on the set

$$R = \{(x, y) : -2 \leq x \leq 2, -1 \leq y \leq 1\}.$$

14. $\boxed{\text{10 pts.}}$ Evaluate $\iint_R e^{x+2y}\, dA$ over the region

$$R = \{(x, y) : 0 \leq x \leq \ln 2, 1 \leq y \leq \ln 3\}$$

15. $\boxed{\text{10 pts.}}$ Evaluate $\iint_R y^3 \sin(xy^2)\, dA$ over the region

$$R = \left\{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq \sqrt{\pi/2}\,\right\},$$

choosing a convenient order.

16. $\boxed{\text{10 pts.}}$ Evaluate $\iint_R (x + y)\, dA$, where R is the region in the first quadrant bounded by $x = 0$, $y = x^2$, and $y = 8 - x^2$.

17. $\boxed{\text{10 pts.}}$ The integral

$$\int_0^{1/2} \int_{y^2}^{1/4} y \cos(16\pi x^2)\, dx dy$$

can only be evaluated by reversing the order of integration. So reverse the order of integration and evaluate.

18. $\boxed{\text{10 pts.}}$ Sketch the region

$$R = \{(x, y) : x^2 + y^2 \leq 9,\ y \geq 0\},$$

then evaluate the integral $\iint_R 2xy\, dA$ using polar coordinates.

19. $\boxed{\text{10 pts.}}$ Use integration to find the area of the region bounded by all leaves of the rose $r = 2\cos 3\theta$.