

MATH 242
SUMMER 2013
EXAM 1

NAME:

1. 10 pts. Given points $p = (-4, 1)$ and $q = (3, -7)$, find two vectors parallel to \overrightarrow{qp} with length 6.
2. 10 pts. Professor Chaos pulls east on a chain attached to an ATM with a force of 200 N, and his partner in crime General Disarray pulls south on a rope attached to the same ATM with a force of 150 N. What is the magnitude and direction of the force on the ATM? Assume all vectors lie in a plane.
3. 10 pts. Having busted out of the Big House, Professor Chaos paddles a canoe due west at 8 km/h relative to the water in a current that flows northwest at 3 km/h. Find the speed and direction of the canoe relative to the shore.
4. 10 pts. Find an equation of the sphere passing through $p = (-4, 2, 3)$ and $q = (0, 2, 7)$ with its center at the midpoint of \overline{pq} .
5. 10 pts. each Let $\mathbf{u} = \langle 2, -1, 5 \rangle$ and $\mathbf{v} = \langle -1, 1, 1 \rangle$.
 - (a) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
 - (b) Find $\text{proj}_{\mathbf{v}} \mathbf{u}$, the orthogonal projection of \mathbf{u} onto \mathbf{v} .
 - (c) Find the angle between \mathbf{u} and \mathbf{v} to the nearest tenth of a degree.
6. 10 pts. Find a vector that is normal to $\langle 6, -2, 4 \rangle$ and $\langle 1, 2, 3 \rangle$.
7. 10 pts. Find the point, if any, where the plane $y = -2$ intersects with the line
$$\mathbf{r}(t) = \langle 2t + 1, -t + 4, t - 6 \rangle, \quad -\infty < t < \infty.$$
8. 10 pts. Find a parameterization for the line containing points $(1, 0, 1)$ and $(3, -3, 3)$.
9. 10 pts. Find the unit tangent vector for the parameterized curve

$$\mathbf{r}(t) = \left\langle t, 2, \frac{2}{t} \right\rangle, \quad t \geq 1$$

at the point corresponding to $t = 2$.

10. 10 pts. Evaluate the definite integral

$$\int_0^1 \langle e^{2t}, e^{-t}, t \rangle dt$$

11. 10 pts. The acceleration of an object at time t is $\mathbf{a}(t) = \langle 1, t \rangle$. Given that the object's initial velocity is $\mathbf{v}(0) = \langle 2, -1 \rangle$ and initial position is $\mathbf{r}(0) = \langle 0, 8 \rangle$, find the object's position at time t .

12. 10 pts. Find the length of the curve $\mathbf{r}(t) = \cos^3 t \mathbf{i} + \sin^3 t \mathbf{j}$, $0 \leq t \leq \pi/2$.

13. 10 pts. Recall the hyperbolic functions $\cosh t = \frac{1}{2}(e^t + e^{-t})$ and $\sinh t = \frac{1}{2}(e^t - e^{-t})$. Find the curvature κ for the curve given by

$$\mathbf{r}(t) = \langle \cosh t, \sinh t, t \rangle.$$

The recommended formula is

$$\kappa(t) = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}.$$

14. 10 pts. each

- (a) Find the curvature of $f(x) = e^x$.
(b) Find the point where the curvature is maximum.

15. 10 pts. Find an equation of the plane containing the points $(1, 1, 0)$, $(-2, 8, 4)$ and $(1, 2, 3)$.

16. 10 pts. Find an equation of the line where the planes $x + 2y - 3z = 1$ and $x + y + z = 2$ intersect.