MATH 242
FALL 2018
EXAM 4

NAME:

1. $\boxed{\text{10 pts.}}$ Find the volume of the region in space bounded by the graphs of $z = 9 - x^2$, $y = -x + 2$, $y = 0$, and $z = 0$, with $x \geq 0$.

2. $\boxed{\text{15 pts.}}$ Rewrite the integral

$$\int_0^1 \int_{-2}^2 \int_0^{\sqrt{4-y^2}} dz\, dy\, dx$$

in the order $dy\, dz\, dx$, then evaluate the resulting integral.

3. $\boxed{\text{15 pts.}}$ Use cylindrical coordinates to find the volume of the region that is inside both the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 2$.

4. $\boxed{\text{15 pts.}}$ Evaluate

$$\iiint_D (x^2 + y^2)\, dV,$$

where D is the region outside the sphere $x^2 + y^2 + z^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 16$.

5. $\boxed{\text{15 pts.}}$ Find the points (if any) where the vector field $\mathbf{F}(x, y) = \langle 2x, y \rangle$ is tangent or normal to the ellipse

$$\frac{x^2}{4} + y^2 = 1.$$

6. Let C be the line segment from $(0, -3, 2)$ to $(1, -7, 4)$.

   (a) $\boxed{\text{5 pts.}}$ Find a parametric description for C in the form $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$.

   (b) $\boxed{\text{10 pts.}}$ Evaluate the line integral

$$\int_C (xz - y^2)\, ds.$$

7. $\boxed{\text{10 pts.}}$ Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle e^{x-1}, xy \rangle$, and the curve C is given by $\mathbf{r}(t) = \langle t^2, t^3 \rangle$ for $0 \leq t \leq 2$.

8. $\boxed{\text{10 pts.}}$ Let $\mathbf{F} = \langle y - x, x \rangle$, and let C be the curve with parametrization $\mathbf{r}(t) = \langle 2\cos t, 2\sin t \rangle$ for $t \in [0, 2\pi]$. Find the flux of $\mathbf{F}$ across C.