

1. 10 pts. Find an equation of the plane containing the points  $(1, 1, 0)$ ,  $(-2, 8, 4)$  and  $(1, 2, 3)$ .
2. 10 pts. Find an equation of the line where the planes  $x + 2y - 3z = 1$  and  $x + y + z = 2$  intersect.
3. 10 pts. Determine at what points in  $\mathbb{R}^2$  the function

$$F(x, y) = \sqrt{x} + \sqrt{1 - x^2 - y^2}$$

is continuous.

4. 10 pts. Graph two level curves of the function  $z = \sqrt{x^2 + 4y^2}$ , labeling each curve with its  $z$ -value.
5. 10 pts. Evaluate the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}.$$

6. 10 pts. Use the Two-Path Test to prove that the limit

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2 \cos y}{x^2 + y^4}$$

does not exist.

7. 10 pts. each Find the partial derivatives indicated.

(a) Given  $g(x, y) = x \ln(x^2 + y^2)$ , find  $g_x$  and  $g_y$ .

(b) Given  $h(x, y, z) = \cos(x + 2y + 3z)$ , find  $h_z$  and  $h_{xy}$ .

8. Let

$$f(x, y) = \begin{cases} -\frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

(a) 10 pts. Is  $f$  continuous at  $(0, 0)$ ? If not, prove it.

(b) 5 pts. Is  $f$  differentiable at  $(0, 0)$ ? If not, why not?

(c) 10 pts. Evaluate  $f_x(0, 0)$ , if it exists.

9. 10 pts. Given  $w = \cos(2x) \sin(3y)$  with  $x = t/2$  and  $y = t^4$ , use an appropriate chain rule to find  $dw/dt$ . Express the answer in terms of  $t$ .

10. Let  $f(x, y) = 2y - 3x^3$ .

- (a) 5 pts. Find the gradient of  $f$ .
- (b) 5 pts. Find the unit vectors that give the direction of steepest ascent and steepest descent at  $(1, 2)$ .
- (c) 10 pts. Let  $C$  be the path of steepest descent on the surface  $z = f(x, y)$  beginning at  $(1, 2, 1)$ , and let  $C_0$  be the projection of  $C$  onto the  $xy$ -plane. Find an equation for  $C_0$ .

11. 10 pts. Compute the directional derivative of

$$f(x, y) = e^x \sin y$$

at the point  $(0, \pi/4)$  in the direction  $\langle 1, \sqrt{3} \rangle$ .