MATH 242
FALL 2017
EXAM 2

NAME:

1. $\boxed{\text{10 pts.}}$ Find an equation for the line of intersection of the planes $x + 2y - z = 1$ and $x + y + z = 2$.

2. $\boxed{\text{10 pts.}}$ Find the domain of the function $\varphi(x, y) = \ln(2y - x^2)$, and make a sketch of the set.

3. $\boxed{\text{15 pts.}}$ For the function

$$F(x, y) = \frac{4x}{x^2 + y^2},$$

graph the level curves $F(x, y) = c$ for $c = \pm\frac{1}{2}, \pm 1$.

4. $\boxed{\text{10 pts.}}$ Evaluate the limit or show that it does not exist:

$$\lim_{(x,y)\to(1,1)} \frac{x^2 + xy - 2y^2}{2x^2 - xy - y^2}.$$

5. $\boxed{\text{10 pts.}}$ Use the Two-Path Test to prove that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}.$$

6. $\boxed{\text{10 pts.}}$ For $z = e^x \sin y$ find z_x, z_y, z_{xx}, and z_{yy}. What is $z_{xx} + z_{yy}$?

7. Let

$$\psi(x, y) = \begin{cases} \dfrac{8xy^2}{x^3 + 2y^3}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(a) $\boxed{\text{10 pts.}}$ Evaluate $\psi_x(0, 0)$ and $\psi_y(0, 0)$, if they exist.

(b) $\boxed{\text{10 pts.}}$ Prove or disprove that ψ continuous at $(0, 0)$.

(c) $\boxed{\text{5 pts.}}$ Prove or disprove that ψ differentiable at $(0, 0)$.

8. Let $f(x, y) = x^2 + 4xy - y^3$, and let $p = (-2, 3)$.

(a) $\boxed{\text{5 pts.}}$ Find the gradient of f at p.

(b) $\boxed{\text{10 pts.}}$ At p, find the unit vectors that point in the directions of steepest ascent, steepest descent, and no change.

9. $\boxed{\text{20 pts.}}$ Let $T(x, y) = 400 - 2x^2 - y^2$ give the temperature in $\mathbb{R}^2$ at the point (x, y). Find a parametrization $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ for the path in $\mathbb{R}^2$ followed by a heat-seeking microbe placed at the point $(10, 10)$.