MATH 242
FALL 2017
EXAM 1

NAME:

1. A small plane is flying north in calm air at 300 km/h when it is hit by a horizontal crosswind blowing northeast at 55 km/h and a 32 km/h downdraft.

   (a) [10 pts.] Find the resulting velocity and speed of the plane.

   (b) [5 pts.] To the nearest degree, what is the angle (with respect to the horizontal) of the plane's descent?

2. [10 pts.] Let $\mathbf{u} = \langle 1, 1\rangle$ and $\mathbf{v} = \langle -2, 1\rangle$. For arbitrary numbers a and b, express $\langle a, b\rangle$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

3. [10 pts.] Find an equation of the sphere passing through $p = (-4, 2, 3)$ and $q = (0, 2, 7)$ with its center at the midpoint of $\overline{pq}$.

4. [10 pts.] Find the values of x and y such that the points $(1, 2, 3)$, $(4, 7, 1)$ and $(x, y, 9)$ are collinear.

5. [10 pts. each] Let $\mathbf{u} = \langle 2, -1, 0\rangle$ and $\mathbf{v} = \langle 4, -8, 3\rangle$.

   (a) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ to the nearest tenth of a degree.

   (b) Find $\text{proj}_{\mathbf{v}}\,\mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.

6. [10 pts.] Find the area of the parallelogram that has two adjacent sides given by $\mathbf{u} = \langle -3, 0, 2\rangle$ and $\mathbf{v} = \langle 1, 1, 1\rangle$.

7. [10 pts.] Find a parametrization for the line through $(0, 2, 1)$ that is perpendicular to both the z-axis and $\mathbf{u} = \langle 4, 3, -5\rangle$.

8. [10 pts.] Determine whether the lines are parallel, intersecting, or skew:

$$\mathbf{r}(t) = \langle 5 + 2t, 3 + 3t, 1 - t\rangle, \quad \mathbf{R}(s) = \langle 13 - 3s, 13 - 4s, 4 - 2s\rangle.$$

   If they intersect, find the point(s) of intersection.

9. [10 pts.] Given that $\mathbf{r}'(t) = \langle e^{2t}, 1 - 2e^{-t}, 1 - 2e^{t}\rangle$ and $\mathbf{r}(0) = \langle 1, 1, 1\rangle$, find the function $\mathbf{r}$.

10. [10 pts.] Find the speed associated with the trajectory

$$\mathbf{r}(t) = \langle 5\cos t^2, 5\sin t^2, 12t^2\rangle,$$

    and then find the length of the trajectory on the interval $0 \le t \le 2$.

11. [15 pts.] Find the curvature κ of the curve $\mathbf{r}(t) = \langle t, t^2, t^2/2\rangle$, and then find the point on the curve where the curvature attains a maximum. What is the maximum curvature value?