Math 242
Fall 2016 Name:
Exam 4

1. $\boxed{\text{10 pts.}}$ Find the volume of the region in space bounded by the graphs of $z = 9 - x^2$, $y = -x + 2$, $y = 0$, and $z = 0$, with $x \geq 0$.

2. $\boxed{\text{15 pts.}}$ Rewrite the integral

$$\int_0^1 \int_{-2}^2 \int_0^{\sqrt{4-y^2}} dz\,dy\,dx$$

in the order $dy\,dz\,dx$, then evaluate the resulting integral.

3. $\boxed{\text{10 pts.}}$ Find the mass of the solid cone

$$D = \{(r, \theta, z) : z \in [0, 6 - r],\ r \in [0, 6]\}$$

with density $\rho(r, \theta, z) = 8 - z$.

4. Let D be the region in $\mathbb{R}^3$ bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 25$.
   (a) $\boxed{\text{3 pts.}}$ Sketch the region D.
   (b) $\boxed{\text{6 pts.}}$ Set up a triple integral to find the volume of D.
   (c) $\boxed{\text{6 pts.}}$ Evaluate the integral.

5. $\boxed{\text{15 pts.}}$ Find the points where the vector field $\mathbf{F}(x, y) = \langle y, x \rangle$ is tangent and normal to the curve

$$C = \{(x, y) : x^2 + y^2 = 1\}.$$

6. Let C be the line segment from $(0, -3, 2)$ to $(1, -7, 4)$.
   (a) $\boxed{\text{5 pts.}}$ Find a parametric description for C in the form $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$.
   (b) $\boxed{\text{10 pts.}}$ Evaluate the line integral

$$\int_C (xz - y^2)\, ds.$$

7. $\boxed{\text{10 pts.}}$ Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle e^{x-1}, xy \rangle$, and the curve C is given by $\mathbf{r}(t) = \langle t^2, t^3 \rangle$ for $0 \leq t \leq 1$.

8. $\boxed{\text{10 pts.}}$ Let $\mathbf{F} = \langle y - x, x \rangle$, and let C be the curve with parametrization $\mathbf{r}(t) = \langle 2\cos t, 2\sin t \rangle$ for $t \in [0, 2\pi]$. Find the flux of $\mathbf{F}$ across C.