Math 242
Fall 2016 Name:
Exam 2

1. [10 pts.] Find an equation of the plane that is parallel to the vectors $\langle 1, -3, 4\rangle$ and $\langle 4, 0, -2\rangle$, passing through the point $(1, 0, 1)$.

2. [10 pts.] For the quadric surface

$$x^2 + \frac{y^2}{4} = z^2,$$

find the equations of the xy-, xz-, and yz-traces, when they exist.

3. [10 pts.] Find the domain of the function

$$\varphi(x, y) = \sqrt{2x - 3y - 1}.$$

Make a sketch of the set.

4. [10 pts.] For the function

$$F(x, y) = \frac{x}{x^2 + y^2},$$

graph the level curves $F(x, y) = c$ for $c = \pm\frac{1}{2}, \pm 1$.

5. [10 pts.] Evaluate the limit or show that it does not exist:

$$\lim_{(x,y)\to(1,1)} \frac{x^2 + xy - 2y^2}{2x^2 - xy - y^2}.$$

6. [10 pts.] Use the Two-Path Test to prove that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}.$$

7. [10 pts.] Find the first partial derivatives of $\psi(t, x) = x^2 \sec(t^3 x)$.

8. Let

$$\psi(x, y) = \begin{cases} \dfrac{5x^2 y}{x^3 + y^3}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

(a) [8 pts.] Evaluate $\psi_x(0, 0)$ and $\psi_y(0, 0)$, if they exist.

(b) [8 pts.] Prove or disprove that ψ continuous at $(0, 0)$.

(c) [4 pts.] Prove or disprove that ψ differentiable at $(0, 0)$.

9. 10 pts. each Let $f(x, y) = x^2 + 4xy - y^3$, and let $p = (3, -1)$.

(a) Find the gradient of f at p.

(b) At p, find the unit vectors that point in the directions of steepest ascent, steepest descent, and no change.

10. 15 pts. Let

$$T(x, y) = 400 - 2x^2 - y^2$$

give the temperature in $\mathbb{R}^2$ at the point (x, y). Find a parametrization $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ for the path in $\mathbb{R}^2$ followed by a heat-seeking microbe placed at the point $(10, 10)$.