MATH 242
FALL 2016
EXAM 1

NAME:

1. [10 pts.] Find two vectors in $\mathbb{R}^2$ that are parallel to $\mathbf{u} = \langle 2, -5 \rangle$, but with 7 times the magnitude.

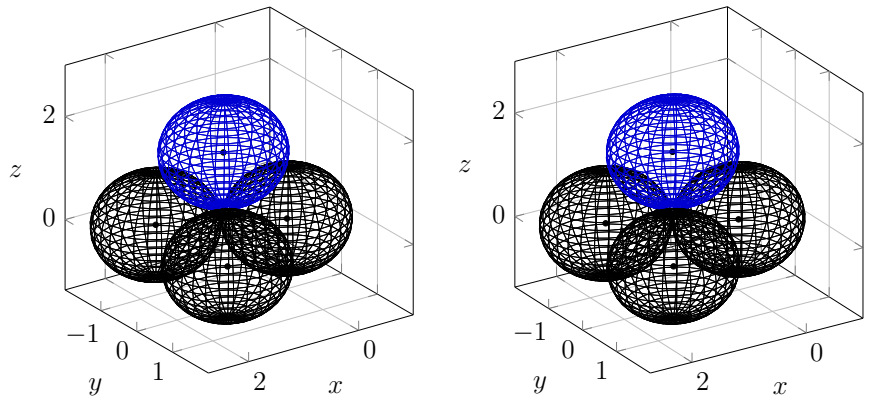
2. [10 pts.] A 100 kg load rests on a flat surface that makes an angle of 30° with the floor. Find the components of the force of gravity acting on the load that are perpendicular and parallel to the surface. (On Earth gravity exerts a force having magnitude mg on an object with mass m, where $g = 9.8$ m/s^2.)

3. [10 pts.] Determine the values of x and y such that the points $(1, 2, 3)$, $(4, 7, 1)$, and $(x, y, 2)$ are collinear (i.e. they lie on a line).

4. Pictured below are four spheres (it is a stereoscopic figure but the depth effect is not needed). The three lower spheres are centered at points $O = (0, 0, 0)$, $P = (\sqrt{3}, -1, 0)$, and $Q = (\sqrt{3}, 1, 0)$. The top sphere sits upon the lower spheres such that its center C is equidistant from O, P, and Q.

   (a) [10 pts.] Find the coordinates of C.

   (b) [5 pts.] Let $\mathbf{r}_{IJ}$ denote, in general, the vector from point I to point J. Find $\mathbf{r}_{PQ}$ and $\mathbf{r}_{PC}$.



5. [10 pts. each] Let $\mathbf{u} = \langle 2, -1, 9 \rangle$ and $\mathbf{v} = \langle -2, -5, 3 \rangle$.

   (a) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ to the nearest tenth of a degree.

   (b) Find $\text{proj}_{\mathbf{v}} \mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.

6. [10 pts.] Find the area of the parallelogram that has two adjacent sides given by $\mathbf{u} = \langle -3, 0, 2 \rangle$ and $\mathbf{v} = \langle 1, 1, 1 \rangle$.

7. [10 pts.] Find a parametrization for the line through the point $(-3, 4, 2)$ that is perpendicular to both $\mathbf{u} = \langle 1, 1, -5 \rangle$ and $\mathbf{v} = \langle 0, 4, 0 \rangle$.

8. [10 pts.] Find a parametrization for the tangent line to the curve

$$\mathbf{r}(t) = \langle 3t - 1, 7t + 2, t^2 \rangle$$

at the point corresponding to $t = 1$, making sure the orientation of the line is the same as the direction of the tangent vector at that point.

9. [10 pts.] Find the length of the curve

$$\mathbf{r}(t) = \langle \cos t + \sin t, \cos t - \sin t \rangle, \quad t \in [0, 2\pi].$$

10. [15 pts.] Find the curvature κ of the helix

$$\mathbf{r}(t) = \langle a \cos t, a \sin t, bt \rangle,$$

where $a, b > 0$ are constants. What happens to the curvature as $b \to \infty$?