MATH 242
FALL 2015
EXAM 3

NAME:

1. $\boxed{\text{10 pts.}}$ Find an equation of the plane tangent to the surface given by $z = \tan^{-1}(xy)$ at the point $(1, 1, \pi/4)$.

2. $\boxed{\text{15 pts.}}$ Find the critical points of $f(x, y) = x^3 - y^3 + 6xy$, then determine whether each critical point corresponds to a local maximum, local minimum, or saddle point.

3. $\boxed{\text{15 pts.}}$ Find the absolute maximum and minimum values of the function

$$f(x, y) = 2x^2 - 4x + 3y^2 + 2$$

on the disc $R = \{(x, y) : (x - 1)^2 + y^2 \leq 1\}$.

4. $\boxed{\text{15 pts.}}$ Use the Method of Lagrange Multipliers to find the maximum and minimum values of $f(x, y) = x^2 + y^2$ subject to the constraint $x^6 + y^6 = 1$.

5. $\boxed{\text{10 pts.}}$ Evaluate the double integral

$$\iint_R \frac{y}{1 + x^2}\, dA,$$

where R is the region bounded by $y = 0$, $y = \sqrt{x}$, and $x = 4$.

6. $\boxed{\text{10 pts.}}$ Use a double integral to find the volume of the region bounded by the coordinate planes $x = 0$, $y = 0$, $z = 0$ and the plane $z = 12 - 2x - 3y$.

7. $\boxed{\text{10 pts.}}$ Use a double integral to find the area of the region R bounded by the parabola $y = x^2$ and the line $y = x + 2$.

8. $\boxed{\text{10 pts.}}$ Sketch the region

$$R = \{(x, y) : x^2 + y^2 \leq 4,\ x \geq 0\, y \geq 0\},$$

then use polar coordinates to evaluate the integral

$$\iint_R \frac{1}{\sqrt{16 - x^2 - y^2}}\, dA.$$

9. $\boxed{\text{10 pts.}}$ Use a double integral and polar coordinates to find the area of the region bounded by all the leaves of the rose $r = 4\cos 3\theta$.