Math 242
Fall 2015
Exam 2

Name:

1. $\boxed{\text{10 pts.}}$ Find a parametrization for the line of intersection of the planes $x+2y-z=1$ and $x+y+z=4$.

2. $\boxed{\text{10 pts.}}$ Find the domain and range of the function

$$f(x,y) = -\frac{12}{\sqrt{49-x^2-y^2}}.$$

Give a geometrical description of the domain.

3. $\boxed{\text{10 pts.}}$ Determine the set of points in $\mathbb{R}^2$ where the function $h(x,y)=\sqrt{x-y^2}$ is continuous. Make a sketch of the set.

4. $\boxed{\text{10 pts.}}$ Graph two level curves of the function $z=\sqrt{x^2+4y^2}$, labeling each curve with its z-value.

5. $\boxed{\text{10 pts.}}$ Evaluate the limit

$$\lim_{(x,y)\to(-1,1)} \frac{x^2-y^2}{x^2-xy-2y^2}.$$

6. $\boxed{\text{10 pts.}}$ Use the Two-Path Test to prove that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}.$$

7. $\boxed{\text{10 pts.}}$ Find the first partial derivatives of $\psi(t,x)=x^2\sec(t^3x)$.

8. Let

$$g(x,y) = \begin{cases} \dfrac{3x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$$

   (a) $\boxed{\text{5 pts.}}$ Evaluate $g_x(0,0)$ and $g_y(0,0)$, if they exist.

   (b) $\boxed{\text{5 pts.}}$ Prove or disprove that g continuous at $(0,0)$.

   (c) $\boxed{\text{5 pts.}}$ Prove or disprove that g differentiable at $(0,0)$.

9. $\boxed{\text{10 pts. each}}$ Let $f(x,y)=x^2+4xy-y^2$, and let $p=(3,-2)$.

   (a) Find the gradient of f at p.

   (b) At p, find the unit vectors that point in the directions of steepest ascent, steepest descent, and no change.

10. $\boxed{\text{15 pts.}}$ Let $f(x,y)=xy$. Let C be the path of steepest descent on the surface $z=f(x,y)$ beginning at $(1,2,2)$, and let C_0 be the projection of C onto the xy-plane. Find a parameterization for C_0 in the form of a vector-valued function $\mathbf{r}(t)=\langle x(t),y(t)\rangle$.