MATH 242
FALL 2014 NAME:
EXAM 4

1. [10 pts.] Find the volume of the region in the first octant bounded by $y = 2x^2$ and $y + 4z = 8$.

2. [10 pts.] Evaluate the integral
$$\int_1^{\ln 8} \int_1^{\sqrt{z}} \int_{\ln y}^{\ln 2y} e^{x+y^2-z}\,dx\,dy\,dz$$

3. [10 pts.] Evaluate the integral
$$\iiint_D \sqrt{x^2+y^2}\,dV$$
using cylindrical coordinates, where D is the region inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

4. Let C be the line segment from $(0, 1, 2)$ to $(-3, 7, -1)$.
   (a) [5 pts.] Find a parametric description for C in the form $\mathbf{r}(t) = \langle x(t), y(t), z(t)\rangle$.
   (b) [10 pts.] Evaluate the line integral
$$\int_C (xz - y^2)\,ds.$$

5. [10 pts.] Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle e^{x-1}, xy\rangle$, and the curve C is given by $\mathbf{r}(t) = \langle t^2, t^3\rangle$ for $0 \le t \le 1$.

6. [10 pts.] Determine whether the vector field
$$\mathbf{F}(x, y) = \langle ye^x + \sin y, e^x + x\cos y\rangle$$
is conservative. If it is, determine a potential function for $\mathbf{F}$.

7. [10 pts.] Evaluate
$$\int_C \nabla(e^{-x}\cos y) \cdot d\mathbf{r},$$
where C is the line segment from $(0, 0)$ to $(\ln 2, 2\pi)$.