MATH 242
FALL 2014 NAME:
EXAM 1

1. [10 pts.] Find all values of a such that $\mathbf{w} = a\mathbf{i} - \frac{1}{4}a\mathbf{j}$ is a unit vector.

2. [10 pts.] Three forces with magnitudes of 400 newtons, 280 newtons, and 350 newtons act on an object at angles of $-30°$, $45°$, and $135°$ with the positive x-axis, respectively. Find the magnitude and direction of the resultant force $\mathbf{F}$.

3. [10 pts.] A remote sensing probe falls vertically with a terminal velocity of 60 m/s when it encounters a horizontal crosswind blowing north at 4 m/s and an updraft blowing vertically at 10 m/s. Find the magnitude and direction of the resulting velocity $\mathbf{v}$ relative to the ground.

4. [10 pts.] Give a geometric description of the set of points $(x, y, z) \in \mathbb{R}^3$ that satisfy the equation

$$x^2 + y^2 + z^2 - 8x + 14y - 18z \geq 65.$$

5. [5 pts. each] Let $\mathbf{u} = \langle 2, -1, 8\rangle$ and $\mathbf{v} = \langle -2, 4, -3\rangle$.
   (a) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
   (b) Find $\text{proj}_{\mathbf{v}}\,\mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.
   (c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ to the nearest tenth of a degree.

6. [10 pts.] Let $\mathbf{v} = \langle 0, 2, 0\rangle$. Give a description of all position vectors $\mathbf{u}$ such that

$$\text{proj}_{\mathbf{v}}(\mathbf{u}) = \text{proj}_{\mathbf{v}}\langle 1, 2, 4\rangle.$$

7. [10 pts.] Find the points, if any, where the plane $y = 1$ intersects with the curve

$$\mathbf{r}(t) = \langle 10\cos t, 2\sin t, 1\rangle, \quad t \in [0, 2\pi].$$

8. [10 pts.] Find a parameterization for the line segment having endpoints $(-1, -8, 4)$ and $(-9, 5, -3)$.

9. [10 pts.] Find the unit tangent vector for the parameterized curve

$$\mathbf{r}(t) = \left\langle t, 2, \frac{2}{t}\right\rangle, \quad t \geq 1$$

at the point corresponding to $t = 2$.

10. [10 pts.] Evaluate the definite integral

$$\int_0^1 \langle e^{2t}, e^{-t}, t \rangle \, dt$$

11. [10 pts.] The acceleration of an object at time t is $\mathbf{a}(t) = \langle 1, t \rangle$. Given that the object's initial velocity is $\mathbf{v}(0) = \langle 2, -1 \rangle$ and initial position is $\mathbf{r}(0) = \langle -3, 6 \rangle$, find the object's position at time t.

12. Let C be the curve in $\mathbb{R}^2$ given by $f(x) = x^3$.

    (a) [5 pts.] Give C using a vector-valued function $\mathbf{r}(t)$.

    (b) [10 pts.] Find the curvature function of C using the formula

$$\kappa(t) = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}.$$

    (c) [10 pts.] Find the point on C where the curvature is greatest, and give the curvature at that point. Do not round your answer!