MATH 242
FALL 2013
EXAM 5

NAME:

1. [10 pts.] Use the circulation form of Green's Theorem to evaluate the line integral $\oint_{\partial R} \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle 2xy, x^2 - y^2 \rangle$ and R is the region bounded by $y = x(2-x)$ and $y = 0$.

2. [10 pts.] Use Green's Theorem to evaluate

$$\int_C f\,dy - g\,dx,$$

where $\langle f, g \rangle = \langle 0, xy \rangle$ and C is the counterclockwise-oriented triangle with vertices $(0,0)$, $(2,0)$, and $(0,4)$.

3. [10 pts.] Find the divergence of $\mathbf{F}(x,y,z) = \langle -2y, 3x, z \rangle$.

4. [10 pts.] Find the curl of $\mathbf{F}(x,y,z) = \langle x^2 - y^2, xy, z \rangle$.

5. Consider the half-cylinder consisting of the set of points

$$\{(r,\theta,z) : r = 4,\ 0 \le \theta \le \pi,\ 0 \le z \le 7\}$$

in polar coordinates.

   (a) [5 pts.] Give a parametric description of the half-cylinder in the form

$$\mathbf{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle.$$

   (b) [10 pts.] Find the area of the half-cylinder by evaluating the appropriate surface integral.

6. [10 pts.] Let $\mathbf{F}(x,y,z) = \langle 2y, -z, x \rangle$, and let C be the circle $x^2 + y^2 = 12$ in the plane $z = 0$ with positive orientation. Evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ by evaluating the surface integral in Stokes' Theorem using an appropriate choice of surface Σ.

7. [10 pts.] Use the Divergence Theorem to find the net outward flux of the field

$$\mathbf{F}(x,y,z) = \langle 2z - y, x, -2x \rangle$$

across the sphere of radius 1 centered at the origin.

8. [10 pts.] The electric field due to a point charge Q is

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0} \frac{\mathbf{r}}{|\mathbf{r}|^3},$$

where $\mathbf{r} = \langle x, y, z \rangle$. Show that the flux of the field across a sphere of radius a centered at the origin is Q/ϵ_0.