Math 242 Fall 2013 Exam 4

## NAME:

1. 10 pts. Find the volume of the wedge of the cylinder  $x^2 + 4y^2 = 16$  created by the planes z = 5 - x and z = x - 5.



2. 10 pts. Evaluate the triple integral

$$\int_{1}^{6} \int_{0}^{4-2y/3} \int_{0}^{12-2y-3z} \frac{1}{y} dx dz dy$$

- 3. 15 pts. Find the volume of the region D bounded by the paraboloid  $z = x^2 + y^2$  and the plane z = 36.
- 4. 10 pts. Evaluate the integral in spherical coordinates:

$$\iiint_D e^{-(x^2+y^2+z^2)^{3/2}} dV,$$

where D is the unit ball.

- 5. <u>5 pts.</u> Specify the component functions of a vector field  $\mathbf{F}$  in  $\mathbb{R}^2$  that is everywhere normal to the line y = 3x.
- 6. Let C be the curve  $y = x^2$  for  $0 \le x \le 8$ .
  - (a) 5 pts. Find a parametric description for C in the form  $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ .
  - (b) 10 pts. Evaluate the line integral

$$\int_C (xy)^{1/3} ds.$$

- 7. 10 pts. Evaluate the integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$ , where  $\mathbf{F}(x, y) = \langle -y, x \rangle$ , and the curve C is the semicircle  $\mathbf{r}(t) = \langle 4 \cos t, 4 \sin t \rangle$  for  $0 \le t \le \pi$ .
- 8. 10 pts. Show that the vector field  $\mathbf{F}(x, y, z) = \langle 2xy^3 z^4, 3x^2 y^2 z^4, 4x^2 y^3 z^3 \rangle$  is conservative on  $\mathbb{R}^3$ , and then determine a potential function.
- 9. 10 pts. Evaluate

$$\int_C \nabla(e^{-x}\cos y) \cdot d\mathbf{r},$$

where C is the line segment from (0,0) to  $(\ln 2, 2\pi)$ .