

1. 5 pts. each Let \mathbf{i} and \mathbf{j} denote the standard unit vectors in \mathbb{R}^2 .
 - (a) Find two unit vectors parallel to $\mathbf{v} = 6\mathbf{i} - 4\mathbf{j}$.
 - (b) Find all values of a such that $\mathbf{w} = a\mathbf{i} - \frac{1}{3}a\mathbf{j}$ is a unit vector.
2. 10 pts. Three forces with magnitudes of 400 newtons, 280 newtons, and 350 newtons act on an object at angles of -30° , 45° , and 135° with the positive x -axis, respectively. Find the magnitude and direction of the resultant force \mathbf{F} .
3. 10 pts. A remote sensing probe falls vertically with a terminal velocity of 60 m/s when it encounters a horizontal crosswind blowing north at 4 m/s and an updraft blowing vertically at 10 m/s. Find the magnitude and direction of the resulting velocity \mathbf{v} relative to the ground.
4. 10 pts. Give a geometric description of the set of points $(x, y, z) \in \mathbb{R}^3$ that satisfy the equation
$$x^2 + y^2 + z^2 - 6x + 6y - 8z - 2 = 0.$$
5. 5 pts. each Let $\mathbf{u} = \langle 2, -1, 5 \rangle$ and $\mathbf{v} = \langle -1, 4, -3 \rangle$.
 - (a) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
 - (b) Find $\text{proj}_{\mathbf{v}} \mathbf{u}$, the orthogonal projection of \mathbf{u} onto \mathbf{v} .
 - (c) Find the angle between \mathbf{u} and \mathbf{v} to the nearest tenth of a degree.
6. 10 pts. Find the area of a parallelogram P , given that three vertices of P are located at $(0, 0, 0)$, $(2, 4, 8)$, and $(1, 4, 10)$.
7. 10 pts. Find the point, if any, where the plane $y = -2$ intersects with the line
$$\mathbf{r}(t) = \langle 2t + 1, -t + 4, t - 6 \rangle, \quad -\infty < t < \infty.$$
8. 10 pts. Find a parameterization for the line containing points $(1, 0, 1)$ and $(3, -3, 3)$.
9. 10 pts. Find the unit tangent vector for the parameterized curve

$$\mathbf{r}(t) = \left\langle t, 2, \frac{2}{t} \right\rangle, \quad t \geq 1$$

at the point corresponding to $t = 2$.

10. 10 pts. Evaluate the definite integral

$$\int_0^1 \langle e^{2t}, e^{-t}, t \rangle dt$$

11. 10 pts. The acceleration of an object at time t is $\mathbf{a}(t) = \langle 1, t \rangle$. Given that the object's initial velocity is $\mathbf{v}(0) = \langle 2, -1 \rangle$ and initial position is $\mathbf{r}(0) = \langle -3, 6 \rangle$, find the object's position at time t .

12. 10 pts. Find the length of the curve $\mathbf{r}(t) = \cos^3 t \mathbf{i} + \sin^3 t \mathbf{j}$, $0 \leq t \leq \pi/2$.

13. 15 pts. Let C be the curve in \mathbb{R}^2 given by $f(x) = x^2$. Find the curvature function of C using the formula

$$\kappa(t) = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}.$$

What is the curvature at $(0, 0)$?