MATH 242
FALL 2012 NAME:
EXAM 5

1. $\boxed{\text{10 pts.}}$ Use the circulation form of Green's Theorem to evaluate the line integral $\oint_{\partial R} \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle 2xy, x^2 - y^2 \rangle$ and R is the region bounded by $y = x(2 - x)$ and $y = 0$.

2. $\boxed{\text{10 pts.}}$ Use the flux form of Green's Theorem to evaluate the line integral $\oint_C \mathbf{F} \cdot \mathbf{n}$, where $\mathbf{F}(x, y) = \langle 0, xy \rangle$ and C is the triangle with vertices $(0, 0)$, $(2, 0)$, $(0, 4)$.

3. $\boxed{\text{10 pts.}}$ Find the divergence of $\mathbf{F}(x, y, z) = \langle yz \sin x, xz \cos y, xy \cos z \rangle$.

4. $\boxed{\text{10 pts.}}$ Find the curl of $\mathbf{F}(x, y, z) = \langle 0, z^2 - y^2, yz \rangle$.

5. Consider the half-cylinder consisting of the set of points

$$\{(r, \theta, z) : r = 4,\ 0 \le \theta \le \pi,\ 0 \le z \le 7\}$$

in polar coordinates.

   (a) $\boxed{\text{5 pts.}}$ Give a parametric description of the half-cylinder in the form

$$\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle.$$

   (b) $\boxed{\text{10 pts.}}$ Find the area of the half-cylinder by evaluating the appropriate surface integral.

6. Let $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$, and let Σ be the cone $z^2 = x^2 + y^2$, $0 \le z \le 1$. Give Σ the orientation for which $\mathbf{n}$ has a positive z-component at each point of Σ where the cone is orientable (which is everywhere but the origin).

   (a) $\boxed{\text{5 pts.}}$ Give a parametric description of Σ in the form

$$\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle.$$

   (b) $\boxed{\text{10 pts.}}$ Find the flux of $\mathbf{F}$ across Σ.

7. $\boxed{\text{10 pts.}}$ Let $\mathbf{F}(x, y, z) = \langle y^2, -z^2, x \rangle$, and let C be the circle $\mathbf{r}(t) = \langle 3 \cos t, 4 \cos t, 5 \sin t \rangle$ for $0 \le t \le 2\pi$. Evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ by evaluating the surface integral in Stokes' Theorem using an appropriate choice of surface Σ.