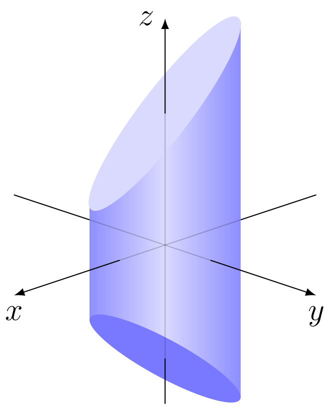
Math 242
Fall 2012
Exam 4

Name:

1. $\boxed{\text{10 pts.}}$ Find the volume of the wedge of the cylinder $x^2 + 4y^2 = 4$ created by the planes $z = 3 - x$ and $z = x - 3$.



2. $\boxed{\text{10 pts.}}$ Evaluate the triple integral

$$\int_1^6 \int_0^{4-2y/3} \int_0^{12-2y-3z} \frac{1}{y} dxdzdy$$

3. $\boxed{\text{15 pts.}}$ Find the volume of the region D bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 25$.

4. $\boxed{\text{10 pts.}}$ Evaluate the integral in spherical coordinates:

$$\iiint_D e^{-(x^2+y^2+z^2)^{3/2}} dV,$$

where D is the unit ball.

5. $\boxed{\text{5 pts.}}$ Specify the component functions of a vector field $\mathbf{F}$ in $\mathbb{R}^2$ that is everywhere normal to the line $x = y$.

6. $\boxed{\text{10 pts.}}$ Evaluate the line integral

$$\int_C (y - z),$$

where C is the helix $\mathbf{r}(t) = \langle 3\cos t, 3\sin t, t\rangle$ for $0 \le t \le 2\pi$.

7. $\boxed{\text{10 pts.}}$ Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle -y, x\rangle$, and the curve C is the semicircle $\mathbf{r}(t) = \langle 4\cos t, 4\sin t\rangle$ for $0 \le t \le \pi$.

8. $\boxed{\text{10 pts.}}$ Show that the vector field $\mathbf{F}(x, y, z) = \langle 2xy^3z^4, 3x^2y^2z^4, 4x^2y^3z^3 \rangle$ is conservative on $\mathbb{R}^3$, and then determine a potential function.

9. $\boxed{\text{10 pts.}}$ Evaluate

$$\int_C \nabla(e^{-x}\cos y) \cdot d\mathbf{r},$$

where C is the line segment from $(0, 0)$ to $(\ln 2, 2\pi)$.