MATH 242
FALL 2012
EXAM 3

NAME:

1. Consider the surface S given by $f(x, y) = (x + y)/(x - y)$.
   (a) [10 pts.] Find an equation of the tangent plane to S at the point $(3, 2, 5)$.
   (b) [5 pts.] Use the tangent plane to estimate the value of $f(2.95, 2.05)$.

2. [15 pts.] Find the critical points of $f(x, y) = xye^{-x-y}$, then determine whether each critical point corresponds to a local maximum, local minimum, or saddle point.

3. [15 pts.] Find the global extrema of the function $f(x, y) = 6 - x^2 - 4y^2$ on the set
$$R = \{(x, y) : -2 \le x \le 2, -1 \le y \le 1\}.$$

4. [10 pts.] Evaluate $\iint_R e^{x+2y}\, dA$ over the region
$$R = \{(x, y) : 0 \le x \le \ln 2, 1 \le y \le \ln 3\}$$

5. [10 pts.] Evaluate $\iint_R y^3 \sin(xy^2)\, dA$ over the region
$$R = \left\{(x, y) : 0 \le x \le 1, 0 \le y \le \sqrt{\pi/2}\right\},$$
choosing a convenient order.

6. [10 pts.] Evaluate $\iint_R (x + y)\, dA$, where R is the region in the first quadrant bounded by $x = 0$, $y = x^2$, and $y = 8 - x^2$.

7. [10 pts.] The integral
$$\int_0^{1/2} \int_{y^2}^{1/4} y \cos(16\pi x^2)\, dx dy$$
can only be evaluated by reversing the order of integration. So reverse the order of integration and evaluate.

8. [10 pts.] Sketch the region
$$R = \{(x, y) : x^2 + y^2 \le 9,\ y \ge 0\},$$
then evaluate the integral $\iint_R 2xy\, dA$ using polar coordinates.

9. [10 pts.] Use integration to find the area of the region bounded by all leaves of the rose $r = 2\cos 3\theta$.