Math 242
Fall 2012 Name:
Exam 2

1. $\boxed{\text{10 pts.}}$ Find an equation of the plane containing the points $(1, 1, 0)$, $(-2, 8, 4)$ and $(1, 2, 3)$.

2. $\boxed{\text{10 pts.}}$ Find an equation of the line where the planes $x + 2y - 3z = 1$ and $x + y + z = 2$ intersect.

3. $\boxed{\text{10 pts.}}$ Determine at what points in $\mathbb{R}^2$ the function $h(x, y) = \ln(x^2 - 3y)$ is continuous.

4. $\boxed{\text{10 pts.}}$ Graph two level curves of the function $z = \sqrt{x^2 + 4y^2}$, labeling each curve with its z-value.

5. $\boxed{\text{10 pts.}}$ Evaluate the limit

$$\lim_{(x,y)\to(2,1)} \frac{x^2 - 4y^2}{x - 2y}.$$

6. $\boxed{\text{10 pts.}}$ Use the Two-Path Test to prove that the limit

$$\lim_{(x,y)\to(0,0)} \frac{xy + y^3}{x^2 + y^2}$$

does not exist.

7. $\boxed{\text{10 pts. each}}$ Find the partial derivatives indicated.
   (a) Given $g(x, y) = x \ln(x^2 + y^2)$, find g_x and g_y.
   (b) Given $h(x, y, z) = \cos(x + 2y + 3z)$, find h_z and h_{zy}.

8. Let

$$f(x, y) = \begin{cases} -\dfrac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

   (a) $\boxed{\text{10 pts.}}$ Is f continuous at $(0, 0)$? If not, prove it.
   (b) $\boxed{\text{5 pts.}}$ Is f differentiable at $(0, 0)$? If not, why not?
   (c) $\boxed{\text{10 pts.}}$ If possible, evaluate $f_y(0, 0)$.

9. $\boxed{\text{10 pts.}}$ Given $w = \cos(2x)\sin(3y)$ with $x = t/2$ and $y = t^4$, use an appropriate chain rule to find $w'(t)$. Express the answer in terms of t.

10. $\boxed{\text{10 pts.}}$ Use a chain rule to find z_s and z_t, where $z = xy - 2x + 3y$ with $x = \sin(s)$ and $y = \tan(t)$.

11. Let $f(x, y) = 2y - 3x^3$.

(a) $\boxed{\text{5 pts.}}$ Find the gradient of f.

(b) $\boxed{\text{5 pts.}}$ Find the unit vectors that give the direction of steepest ascent and steepest descent at $(1, 2)$.

(c) $\boxed{\text{10 pts.}}$ Let C be the path of steepest descent on the surface $z = f(x, y)$ beginning at $(1, 2, 1)$, and let C_0 be the projection of C onto the xy-plane. Find an equation for C_0.

12. $\boxed{\text{10 pts.}}$ Compute the directional derivative of

$$f(x, y) = e^x \sin y$$

at the point $(0, \pi/4)$ in the direction $\langle 1, \sqrt{3}\, \rangle$.