Math 242 Fall 2012 Exam 1

NAME:

- 1. 10 pts. Given points p = (-4, 1) and q = (3, -7), find two vectors parallel to \vec{qp} with length 6.
- 2. 10 pts. Professor Chaos pulls east on a chain attached to an ATM with a force of 200 N, and his partner in crime General Disarray pulls south on a rope attached to the same ATM with a force of 150 N. What is the magnitude and direction of the force on the ATM? Assume all vectors lie in a plane.
- 3. 10 pts. Having busted out of the Big House, Professor Chaos paddles a canoe due west at 8 km/h relative to the water in a current that flows northwest at 3 km/h. Find the speed and direction of the canoe relative to the shore.
- 4. 10 pts. Find an equation of the sphere passing through p = (-4, 2, 3) and q = (0, 2, 7) with its center at the midpoint of \overline{pq} .
- 5. 10 pts. Given vectors $\mathbf{u} = \langle -1, 1, 0 \rangle$ and $\mathbf{v} = \langle 2, -4, 1 \rangle$, find $|\mathbf{u} + 3\mathbf{v}|$.
- 6. 10 pts. Given vectors $\mathbf{u} = \langle 3, -5, 2 \rangle$ and $\mathbf{v} = \langle -9, 5, 1 \rangle$, calculate proj_v \mathbf{u} .
- 7. 10 pts. Find a vector that is normal to (6, -2, 4) and (1, 2, 3).
- 8. 10 pts. Find the point, if any, where the plane y = -2 intersects with the line $\mathbf{r}(t) = \langle 2t+1, -t+4, t-6 \rangle, -\infty < t < \infty.$
- 9. 10 pts. Find an equation for the line through (1, 0, 1) and (3, -3, 3).
- 10. 10 pts. Find the unit tangent vector for the parameterized curve

$$\mathbf{r}(t) = \left\langle t, 2, \frac{2}{t} \right\rangle, \ t \ge 1$$

at the point corresponding to t = 2.

11. 10 pts. Evaluate the definite integral

$$\int_0^{\ln(2)} \left(e^{-t} \mathbf{i} + 2e^{2t} \mathbf{j} - 4e^t \mathbf{k} \right) dt$$

- 12. 10 pts. The acceleration of an object at time t is $\mathbf{a}(t) = \langle 1, t \rangle$. Given that the object's initial velocity is $\mathbf{v}(0) = \langle 2, -1 \rangle$ and initial position is $\mathbf{r}(0) = \langle 0, 8 \rangle$, find the object's position at time t.
- 13. 10 pts. Find the length of the curve $\mathbf{r}(t) = \langle \cos t + t \sin t, \sin t t \cos t \rangle, 0 \le t \le \pi/2.$
- 14. 10 pts. Find the curvature κ for the curve given by $\mathbf{r}(t) = \langle t, 2t^2 \rangle$. The recommended formula is

$$\kappa(t) = \frac{1}{|\mathbf{v}|} \left| \frac{d\mathbf{T}}{dt} \right|.$$

15. 10 pts. each

- (a) Find the curvature of $f(x) = e^x$.
- (b) Find the point where the curvature is maximum.