MATH 242
FALL 2011
EXAM 5

NAME:

1. [10 pts.] Find the gradient field $\mathbf{F} = \nabla\varphi$ for the potential function $\varphi(x, y) = \sin x \sin y$.

2. [10 pts.] Evaluate $\displaystyle\int_C \frac{xy}{z}$, where C is the line segment from $(1, 4, 1)$ to $(3, 6, 3)$.

3. [10 pts.] Compute the flux for the vector field $\mathbf{F}(x, y) = \langle y - x, x\rangle$ across $C : \mathbf{r}(t) = \langle 2\cos t, 2\sin t\rangle$, $0 \le t \le 2\pi$.

4. [15 pts.] Show that the vector field $\mathbf{F}(x, y, z) = \left\langle \frac{1}{y}, -\frac{x}{y^2}, 2z - 1 \right\rangle$ is conservative on $\mathbb{R}^3$, and then determine a potential function.

5. [10 pts.] Evaluate $\displaystyle\int_C \nabla\varphi \cdot d\mathbf{r}$ for $\varphi(x, y, z) = x + y + z$ and $C : \mathbf{r}(t) = \langle \sin t, \cos t, t/\pi\rangle$, $0 \le t \le \pi$.

6. [10 pts.] Use the circulation form of Green's Theorem to evaluate the line integral $\displaystyle\oint_{\partial R} \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = \langle 2xy, x^2 - y^2\rangle$ and R is the region bounded by $y = x(2 - x)$ and $y = 0$.

7. [10 pts.] Use the flux form of Green's Theorem to evaluate the line integral $\displaystyle\oint_C \mathbf{F} \cdot \mathbf{n}$, where $\mathbf{F}(x, y) = \langle 0, xy\rangle$ and C is the triangle with vertices $(0, 0)$, $(2, 0)$, $(0, 4)$.

8. [10 pts.] Find the divergence of $\mathbf{F}(x, y, z) = \langle yz \sin x, xz \cos y, xy \cos z\rangle$.

9. [10 pts.] Find the curl of $\mathbf{F}(x, y, z) = \langle 0, z^2 - y^2, yz\rangle$.

10. [10 pts.] Find all vectors $\mathbf{v}$ for which $(\operatorname{curl} \mathbf{F}) \cdot \mathbf{v} = 0$, given $\mathbf{F}(x, y, z) = \langle y, -2z, -x\rangle$.