MATH 242
FALL 2011
EXAM 4

NAME:

1. $\boxed{\text{10 pts.}}$ Evaluate $\displaystyle\iint_R (x+y)\,dA$, where R is the region in the first quadrant bounded by $x=0$, $y=x^2$, and $y=8-x^2$.

2. $\boxed{\text{10 pts.}}$ The integral $\displaystyle\int_0^{1/2}\int_{y^2}^{1/4} y\cos(16\pi x^2)\,dxdy$ can only be evaluated by reversing the order of integration. So reverse the order of integration and evaluate.

3. $\boxed{\text{10 pts.}}$ Sketch the region $R=\{(x,y) : x^2+y^2\le 9,\ y\ge 0\}$, then evaluate the integral $\displaystyle\iint_R 2xy\,dA$ using polar coordinates.

4. $\boxed{\text{10 pts.}}$ Use integration to find the area of the region bounded by all leaves of the rose $r=2\cos 3\theta$.

5. $\boxed{\text{10 pts.}}$ Evaluate $\displaystyle\iiint_D (xy+xz+yz)\,dV$, where

$$D=\{(x,y,z)\ :\ -1\le x\le 1,\ -2\le y\le 2,\ -3\le z\le 3\}.$$

6. $\boxed{\text{10 pts.}}$ Find the volume of the region bounded by the parabolic cylinder $y=x^2$ and the planes $z=3-y$ and $z=0$.

7. $\boxed{\text{10 pts.}}$ Evaluate in cylindrical coordinates: $\displaystyle\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{4} dz\,dy\,dx.$

8. $\boxed{\text{10 pts.}}$ Evaluate in spherical coordinates: $\displaystyle\int_0^{\pi}\int_0^{\pi/6}\int_{2\sec\varphi}^{4} \rho^2\sin\varphi\,d\rho d\varphi d\theta$

9. $\boxed{\text{10 pts.}}$ Set up a triple integral in spherical coordinates which will determine the volume of the region outside the cone $\varphi=\pi/4$ and inside the sphere $\rho=4\cos\varphi$. Do not evaluate the integral!