1. Let $f(x,y) = 2y - 3x^3$.

   (a) [5 pts.] Find the gradient of f.

   (b) [5 pts.] Find the unit vectors that give the direction of steepest ascent and steepest descent at $(1,2)$.

   (c) [10 pts.] Let C be the path of steepest descent on the surface $z = f(x,y)$ beginning at $(1,2,1)$, and let C_0 be the projection of C onto the xy-plane. Find an equation for C_0.

2. [10 pts.] Compute the directional derivative of $f(x,y) = \ln(4 + x^2 + y^2)$ at the point $(-1,2)$ in the direction $\langle 2, 1 \rangle$.

3. [10 pts.] Given $f(x,y) = 16 - x^2/4 - y^2/16$, find the slope of the tangent line to the level curve $f(x,y) = 12$ at the point $(2\sqrt{3}, 4)$.

4. Consider the surface S given by $f(x,y) = \sqrt{x^2 + y^2}$.

   (a) [10 pts.] Find an equation of the tangent plane to S at the point $(3, -4, 5)$.

   (b) [5 pts.] Use the tangent plane to estimate the value of $f(3.06, -3.92)$.

5. [10 pts.] Find the critical points of $f(x,y) = x^3/3 - y^3/3 + 2xy$, then determine whether each critical point corresponds to a local maximum, local minimum, or saddle point.

6. [15 pts.] Find the global extrema of the function $f(x,y) = 4 + 2x^2 + y^2$ on the set

$$R = \{(x,y) : -1 \le x \le 1, -1 \le y \le 2\}.$$

7. [10 pts.] Evaluate $\iint_R e^{x+2y} \, dA$ over the region $R = \{(x,y) : 0 \le x \le \ln 2, 1 \le y \le \ln 3\}$

8. [10 pts.] Evaluate $\iint_R y^3 \sin(xy^2) \, dA$ over the region $R = \left\{(x,y) : 0 \le x \le 1, 0 \le y \le \sqrt{\pi/2}\right\}$, choosing a convenient order.