Math 242
Fall 2011   **Name:**
Exam 2

1. [10 pts.] Find an equation of the plane containing the points $(1, 1, 0)$, $(-2, 1, 1)$ and $(1, 2, 3)$.

2. [10 pts.] Find an equation of the line where the planes $x + 2y - 3z = 1$ and $x + y + z = 2$ intersect.

3. [10 pts.] Find the domain of $f(x, y) = \sqrt{\dfrac{xy}{x^2 + y^2}}$.

4. [10 pts.] Find the domain and range of $\varphi(x, y) = \sqrt{16 - x^2 - y^2}$.

5. [10 pts.] Graph two level curves of the function $z = \sqrt{y - x^2 - 1}$, labeling each curve with its z-value.

6. [10 pts.] Evaluate $\displaystyle\lim_{(x,y)\to(1,2)} \dfrac{\sqrt{y} - \sqrt{x+1}}{y - x - 1}$.

7. [10 pts.] Use the Two-Path Test to prove that the limit does not exist: $\displaystyle\lim_{(x,y)\to(0,0)} \dfrac{y}{\sqrt{x^2 - y^2}}$.

8. [10 pts.] Evaluate, using limit laws if necessary: $\displaystyle\lim_{(x,y)\to(0,1)} \dfrac{2y \sin(x)}{x(y + 6)}$.

9. [10 pts. each] Find the first-order partial derivatives of each function.

   (a) $g(x, y) = x \ln(x^2 + y^2)$

   (b) $h(x, y, z) = \cos(x + 2y + 3z)$

10. Let
$$f(x, y) = \begin{cases} -\dfrac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

    (a) [10 pts.] Is f continuous at $(0, 0)$? If not, prove it.

    (b) [5 pts.] Is f differentiable at $(0, 0)$? If not, why not?

    (c) [10 pts.] If possible, evaluate $f_y(0, 0)$.

11. [10 pts.] Given $w = \cos(2x)\sin(3y)$ with $x = t/2$ and $y = t^4$, use an appropriate chain rule to find $w'(t)$. Express the answer in terms of t.

12. [10 pts.] Use a chain rule to find z_s and z_t, where $z = xy - 2x + 3y$ with $x = \sin(s)$ and $y = \tan(t)$.