Math 242
Fall 2011
Exam 1

Name:

1. [5 pts.] Define points $p(3,-5)$ and $q(-2,-7)$. Express $\overrightarrow{pq}$ in the form $a\mathbf{i}+b\mathbf{j}$.

2. [10 pts.] A boat is towed with a force of 200 N with a rope that makes an angle of 22° to the horizontal. Find the horizontal and vertical components of the force, rounded to the tenths place.

3. [10 pts.] A woman paddles a canoe due west at 7 km/h relative to the water in a current that flows northwest at 3 km/h. Find the speed and direction of the canoe relative to the shore.

4. [10 pts.] Find an equation of the sphere passing through $p(-4,2,3)$ and $q(0,2,7)$ with its center at the midpoint of $\overline{pq}$.

5. [5 pts. each] Let $\mathbf{v}=\langle 5,-2,4\rangle$.
   (a) Find the magnitude of $\mathbf{v}$.
   (b) Find two unit vectors parallel to $\mathbf{v}$.

6. [10 pts.] Given the vectors $\mathbf{u}=\langle 10,5\rangle$ and $\mathbf{v}=\langle 2,6\rangle$, calculate $\text{proj}_{\mathbf{v}}\,\mathbf{u}$.

7. [10 pts.] Given $\mathbf{u}=\langle 4,3,0\rangle$ & $\mathbf{v}=\langle 1,1,1\rangle$, express $\mathbf{u}$ as the sum $\mathbf{u}=\mathbf{p}+\mathbf{n}$, where $\mathbf{p}$ is parallel to $\mathbf{v}$ and $\mathbf{n}$ is orthogonal to $\mathbf{v}$.

8. [10 pts.] Given $\mathbf{u}=\langle -4,1,1\rangle$ & $\mathbf{v}=\langle 0,1,-1\rangle$, calculate $\mathbf{u}\times\mathbf{v}$.

9. [10 pts.] Find an equation for the line through $(1,0,1)$ and $(3,-3,3)$.

10. [10 pts.] Find the domain of the function

$$\mathbf{r}(t)=\sqrt{9-t^2}\,\mathbf{i}+\sqrt{t}\,\mathbf{j}-\frac{1}{t-1}\,\mathbf{k}$$

11. [10 pts.] Find the unit tangent vector for the parameterized curve

$$\mathbf{r}(t)=\left\langle \sin t,\cos t,\sqrt{t}\right\rangle,\quad 0\le t<\infty$$

at the point corresponding to $t=9$.

12. [10 pts.] Find the function $\mathbf{r}$ for which

$$\mathbf{r}'(t)=\langle\sqrt{t},\cos(\pi t),4/t\rangle$$

and $\mathbf{r}(1)=\langle 2,3,4\rangle$.

13. [10 pts.] Find the length of the curve given by

$$\mathbf{r}(t)=\left\langle \frac{t^2}{2},\frac{(2t+1)^{3/2}}{3}\right\rangle,\ 0\le t\le 2.$$

(This is *not* the "hard" one done in class.)

14. [10 pts.] Find the curvature κ for the curve given by $\mathbf{r}(t)=\langle 2t,4\sin t,4\cos t\rangle$. The recommended formula is

$$\kappa=\frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right|.$$