Math 242
Exam 5
Fall 2010

Name:

1. [10 pts.] Evaluate $\int_C (x^2 + y^2)\, ds$, where C is the line segment from $(0, 0)$ to $(5, 5)$.

2. [10 pts.] Compute the flux for the vector field $\mathbf{F}(x, y) = \langle y - x, x\rangle$ across $C : \mathbf{r}(t) = \langle 2\cos t, 2\sin t\rangle$, $0 \le t \le 2\pi$.

3. [15 pts.] Determine whether the vector field $\mathbf{F}(x, y, z) = \langle y + z, x + z, x + y\rangle$ is conservative on $\mathbb{R}^3$. If it is, determine a potential function.

4. [10 pts.] Evaluate $\int_C \nabla\varphi \cdot d\mathbf{r}$ for $\varphi(x, y, z) = x + y + z$ and $C : \mathbf{r}(t) = \langle \sin t, \cos t, t/\pi\rangle$, $0 \le t \le \pi$.

5. [10 pts.] Use a line integral on the boundary ∂R to find the area of the region R bounded by the parabolas $\mathbf{r}_1(t) = \langle t, 2t^2\rangle$ and $\mathbf{r}_2(t) = \langle t, 12 - t^2\rangle$.

6. [10 pts.] Use Green's Theorem to evaluate $\oint_C \mathbf{F} \cdot \mathbf{n}$, where $\mathbf{F}(x, y, z) = \langle 2x + e^{y^2}, 4y^2 + e^{x^2}\rangle$ and C is the boundary of the square with vertices $(0, 0), (1, 0), (1, 1), (0, 1)$.

7. [5 pts.] Find the divergence of $\mathbf{F}(x, y, z) = \langle e^{-x+y}, e^{-y+z}, e^{-z+x}\rangle$.

8. [10 pts.] Find the curl of $\mathbf{F}(x, y, z) = \langle x^2 - z^2, 1, 2xz\rangle$.

9. [10 pts.] Find all vectors $\mathbf{v}$ for which $(\operatorname{curl}\mathbf{F}) \cdot \mathbf{v} = 0$, given $\mathbf{F}(x, y, z) = \langle 2y, -3z, x\rangle$.