Math 242
Exam 4 **Name:**
Fall 2010

1. $\boxed{\text{10 pts.}}$ Evaluate $\displaystyle\iiint_D (xy + xz + yz)\,dV$, where

$$D = \{(x, y, z) \,:\, -1 \le x \le 1,\ -2 \le y \le 2,\ -3 \le z \le 4\}.$$

2. $\boxed{\text{10 pts.}}$ Find the volume of the region between the sphere $x^2 + y^2 + z^2 = 19$ and the hyperboloid $z^2 - x^2 - y^2 = 1$ for $z > 0$.

3. $\boxed{\text{10 pts.}}$ Evaluate $\displaystyle\int_1^{\ln 8} \int_1^{\sqrt{z}} \int_{\ln y}^{\ln(2y)} e^{x+y^2-z}\,dx\,dy\,dz.$

4. $\boxed{\text{10 pts.}}$ Evaluate in cylindrical coordinates: $\displaystyle\int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{4} dz\,dy\,dx.$

5. $\boxed{\text{10 pts.}}$ Find the mass of the solid cone $D = \{(r, \theta, z) : 0 \le z \le 6 - r, 0 \le r \le 6\}$ with density $\rho(r, \theta, z) = 7 - z$.

6. $\boxed{\text{10 pts.}}$ Use spherical coordinates to find the volume of the region bounded by the sphere $\rho = 2\cos\varphi$ and the hemisphere $\rho = 1$, $z \ge 0$.

7. $\boxed{\text{10 pts.}}$ Find the coordinates of the center of mass of the region $R = \{(x, y) \,:\, 0 \le x \le 4,\ 0 \le y \le 2\}$ with density $\rho(x, y) = 1 + x/2$.

8. $\boxed{\text{10 pts.}}$ Find the gradient field $\mathbf{F} = \nabla\varphi$ for the potential function $\varphi(x, y) = x + y^2$.

9. $\boxed{\text{10 pts.}}$ Show that the vector field $\mathbf{F}$ in Problem 8 is orthogonal to all points (x, y) on the equipotential curve that passes through the point $(1, 1)$.