Math 242
Exam #3
Fall 2010

Name:

1. [10 pts.] Compute the directional derivative of $f(x,y) = \dfrac{x}{x-y}$ at the point $(4,1)$ in the direction $\langle -1, 2\rangle$.

2. [10 pts. each] Consider the function $f(x,y) = 6x^2 + 4xy - 3y^2$ and the point $P(6,-1)$.
   (a) Find the unit vectors that give the direction of steepest ascent and steepest descent at P.
   (b) Find a vector that points in a direction of no change in f at P.

3. [15 pts.] Let $f(x,y) = 4 - x^2 - 2y^2$. Let C' be the path of steepest descent on the surface $z = f(x,y)$ beginning at $(1,1,1)$, and let C be the projection of C' on the xy-plane. Find an equation for C.

4. [10 pts.] Find an equation of the plane tangent to the surface $z = x^2 e^{x-y}$ at the point $(2,2,4)$.

5. [20 pts.] Find the critical points of $f(x,y) = x^4 + 2y^2 - 4xy$. For each critical point, use the Second Derivative Test to determine whether it is a local maximum, local minimum, or saddle point.

6. [10 pts.] Evaluate $\displaystyle\int_0^{\pi/2}\int_0^1 x\cos xy\, dydx$

7. [10 pts.] Evaluate $\displaystyle\iint_{\mathcal{R}} (x^2 + xy)\, dA$ over $\mathcal{R} = \{(x,y) : 1 \le x \le 2,\ -1 \le y \le 1\}$.

8. [10 pts.] Evaluate $\displaystyle\iint_{\mathcal{R}} y^2\, dA$ where $\mathcal{R}$ is bounded by $x = 1$, $y = 2x + 2$, and $y = -x - 1$.

9. [15 pts.] The integral $\displaystyle\int_0^{1/2}\int_{y^2}^{1/4} y\cos(16\pi x^2)\, dxdy$ can only be evaluated by reversing the order of integration. So reverse the order of integration and evaluate.

10. [10 pts.] Use integration to find the area of the region bounded by all leaves of the rose $r = 2\cos 3\theta$.