Math 242
Exam #2 **Name:**
Fall 2010

1. $\boxed{\text{20 pts.}}$ For the parameterized curve given by $\mathbf{r}(t) = \langle 2t, 4\sin t, 4\cos t\rangle$, find the unit tangent vector $\mathbf{T}$, the curvature κ, and the principal unit normal vector $\mathbf{N}$.

2. $\boxed{\text{5 pts.}}$ Find an equation for the plane containing point $(1, 2, -3)$ and having normal vector $\mathbf{n} = \langle -2, 5, -1\rangle$.

3. $\boxed{\text{10 pts.}}$ Find an equation for the line where the planes $x + 2y - z = 1$ and $x + y + z = 1$ intersect.

4. $\boxed{\text{10 pts. each}}$ Find the domain of the function.

   (a) $g(x, y) = \ln(x^2 - y)$

   (b) $h(x, y) = \sqrt{x - 2y + 4}$

5. $\boxed{\text{10 pts. each}}$ Find the limit.

   (a) $\displaystyle\lim_{(x,y,z)\to(1,\ln 2,3)} ze^{xy}$

   (b) $\displaystyle\lim_{(x,y)\to(2,2)} \frac{y^2 - 4}{xy - 2x}$

6. $\boxed{\text{15 pts.}}$ Prove that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{y^3 + x^3}{xy^2}$$

7. $\boxed{\text{10 pts. each}}$ Find the first partial derivatives of the function.

   (a) $f(x, y) = y^2 \tan xy$

   (b) $\rho(u, v, w) = \dfrac{u}{v + 2w}$

8. $\boxed{\text{10 pts. each}}$ Let

$$f(x, y) = \begin{cases} -\dfrac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

   (a) Is f continuous at $(0, 0)$? If not, prove it.

   (b) Is f differentiable at $(0, 0)$? If not, why not?

   (c) If possible, evaluate $f_x(0, 0)$.

9. $\boxed{\text{10 pts.}}$ Using the appropriate chain rule, find z_s and z_t, where $z = \sin x \cos 2y$ with $x = s + t$ and $y = s - t$.

10. $\boxed{\text{10 pts.}}$ Assuming $ye^{xy} - 2 = 0$ implicitly defines y as a differentiable function of x, find dy/dx.