Math 242
Exam #1
Fall 2010

Name:

1. [5 pts.] Define the points $P(-4, 1)$ and $Q(3, -5)$. Express $\overrightarrow{PQ}$ in the form $a\mathbf{i} + b\mathbf{j}$.

2. [10 pts.] Find a vector with direction $\langle -7, 9\rangle$ and magnitude 5.

3. [10 pts.] An airplane is flying horizontally due west at 320 mi/hr in still air. Suddenly it encounters a steady 40-mi/hr wind that blows horizontally toward the southwest. Find the airplane's new speed and direction relative to the ground.

4. [10 pts.] Give a geometric description of the set of points that satisfy the equation

$$x^2 + y^2 + z^2 - 6x + 6y - 8z - 2 = 0.$$

5. [10 pts.] Find the components of the vertical force $\mathbf{F} = \langle 0, -20\rangle$ in the directions parallel to and normal to the plane that makes an angle of $\pi/3$ with the positive x-axis.

6. [10 pts.] Find all vectors $\langle 1, a, b\rangle$ orthogonal to $\langle 4, -8, 2\rangle$.

7. [10 pts.] Find the cross product $\mathbf{u} \times \mathbf{v}$ for the vectors $\mathbf{u} = \langle 3, -4, 6\rangle$ and $\mathbf{v} = \langle 1, 2, -1\rangle$.

8. [10 pts.] Find an equation of the line through $(1, 0, 2)$ and $(3, -2, 3)$.

9. [10 pts.] Find the point(s) (if any) at which the plane $z = 16$ intersects with the curve $\mathbf{r}(t) = \langle t, 2t, 4 + 3t\rangle$, $-\infty < t < \infty$.

10. [10 pts.] Find the unit tangent vector for the parameterized curve

$$\mathbf{r}(t) = \left\langle \sin t, \cos t, \sqrt{t} \right\rangle, \quad 0 \le t < \infty$$

at the point corresponding to $t = 9$.

11. [10 pts.] Find the function $\mathbf{r}$ that satisfies the following conditions: $\mathbf{r}'(t) = \langle \sqrt{t},\ \cos \pi t, 4/t\rangle$ and $\mathbf{r}(1) = \langle 2, 3, 4\rangle$.

12. [15 pts.] The position of planet Ziltoid is given by $\mathbf{r}(t) = \langle 30 \cos 3t, 40 \sin t\rangle$ (Ziltoid exists in a very strange universe where the laws of physics are somewhat different, you see). Find the velocity and speed of the planet, and also find the planet's acceleration.

13. [10 pts.] Find the length of the curve given by $\mathbf{r}(t) = \left\langle \dfrac{t^2}{2}, \dfrac{8(t+1)^{3/2}}{3} \right\rangle$, $0 \le t \le 2$.