Math 141 Quiz #5 (Fall 2020)

1 Find the Taylor series with center at $a = \ln 2$ for $f(x) = e^x$, writing the first four nonzero terms explicitly and then expressing the series in summation notation.

$$2 + 2(x - \ln 2) + (x - \ln 2)^2 + \frac{1}{3}(x - \ln 2)^3 + \cdots = \sum_{n=0}^{\infty} \frac{2}{n!}(x - \ln 2)^n.$$

2 Approximate the value of $\int_0^{0.2} x \ln(1 + x^2)\, dx$ with an absolute error less than 10^{-5}. Note:

$$\ln(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

for $x \in (-1, 1]$.

We have

$$\int_0^{0.2} x \ln(1 + x^2)\, dx = \int_0^{0.2} \left(\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{n} \right) dx = \sum_{n=1}^{\infty} \left(\frac{(-1)^{n+1}}{n} \int_0^{0.2} x^{2n+1} dx \right)$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(0.2)^{2n+2}}{n(2n+2)} = \frac{0.2^4}{4} - \frac{0.2^6}{12} + \frac{0.2^8}{24} - \cdots.$$

Since $0.2^6/12 < 10^{-5}$, the estimate

$$\int_0^{0.2} x \ln(1 + x^2)\, dx \approx \frac{0.2^4}{4} = \frac{1}{2500}$$

will have an absolute error less than 10^{-5}.