MATH 141 QUIZ #3 (FALL 2020)

1 Find a recurrence relation and explicit formula for the sequence $(64, 32, 16, 8, 4, 2, \ldots)$.

Recurrence relation:

$$a_n = \frac{1}{2}a_{n-1}, \quad a_1 = 64.$$

Explicit formula:

$$a_n = 2^{7-n}, \quad n \geq 1.$$

2a Find the limit of the sequence $\left(\left(1 + \frac{4}{n}\right)^{3n}\right)$.

With L'Hôpital's Rule,

$$\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^{3n} = \lim_{n\to\infty} e^{3n\ln(1+4/n)} = \exp\left(\lim_{n\to\infty}\frac{\ln(1+4/n)}{1/(3n)}\right)$$

$$= \exp\left(\lim_{n\to\infty}\frac{\dfrac{-4/n^2}{1+4/n}}{-1/(3n^2)}\right) = \exp\left(\lim_{n\to\infty}\frac{12}{1+4/n}\right) = e^{12}.$$

2b Find the limit of the sequence $\left(n\left(1 - \cos\frac{1}{n}\right)\right)$.

Again with L'Hôpital's Rule,

$$\lim_{n\to\infty}\left(n\left(1-\cos\frac{1}{n}\right)\right) = \lim_{n\to\infty}\frac{1-\cos(1/n)}{1/n} = \lim_{n\to\infty}\frac{-\dfrac{1}{n^2}\sin\dfrac{1}{n}}{-\dfrac{1}{n^2}} = \lim_{n\to\infty}\sin\frac{1}{n} = \sin 0 = 0.$$