Math 141 Quiz #2 (Fall 2020)

1 Find $\displaystyle\int xe^{-5x}\,dx$.

Use integration by parts with $u = x$, $v' = e^{-5x}$, so $u' = 1$, $v = -\frac{1}{5}e^{-5x}$, and the integral becomes

$$\int xe^{-5x}\,dx = -\frac{x}{5}e^{-5x} - \int -\frac{1}{5}e^{-5x}\,dx = -\frac{x}{5}e^{-5x} - \frac{1}{25}e^{-5x} + C.$$

2 Find $\displaystyle\int \sin^3 x \cos^{-2} x\,dx$.

Let $u = \cos x$, so $du = -\sin x\,dx$, and then

$$\begin{aligned}\int \sin^3 x \cos^{-2} x\,dx &= -\int (1-\cos^2 x)\cos^{-2} x \sin x\,dx \\ &= -\int (1-u^2)u^{-2}\,du = -\int (u^{-2}-1)\,du \\ &= \frac{1}{u} + u + C = \sec x + \cos x + C.\end{aligned}$$

3 Find $\displaystyle\int \sqrt{81-x^2}\,dx$.

Let $x = 9\sin\theta$, so $dx = 9\cos\theta\,d\theta$. Then, using a couple trigonometric identities, we obtain

$$\begin{aligned}\int \sqrt{81-x^2}\,dx &= \int \sqrt{81-81\sin^2\theta}\cdot 9\cos\theta\,d\theta = 81\int \sqrt{\cos^2\theta}\cos\theta\,d\theta \\ &= 81\int \cos^2\theta\,d\theta = 81\int \frac{1+\cos 2\theta}{2}\,d\theta \\ &= \frac{81}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C = \frac{81}{2}\theta + \frac{81}{2}\sin\theta\cos\theta \\ &= \frac{81}{2}\sin^{-1}\left(\frac{x}{9}\right) + \frac{x\sqrt{81-x^2}}{2} + C,\end{aligned}$$

noting that $\sin\theta = \dfrac{x}{9}$ and $\cos\theta = \dfrac{\sqrt{81-x^2}}{9}$.