Math 141 Quiz #1 (Fall 2020)

1a Find $\dfrac{d}{dx}(e^x \ln x)$.

With the Product Rule,

$$e^x(\ln x)' + (e^x)' \ln x = \frac{e^x}{x} + e^x \ln x = e^x\left(\frac{1}{x} + \ln x\right).$$

1b Find $\dfrac{d}{dx}\left[(3x)^{-x^2}\right]$.

$$\frac{d}{dx}e^{-x^2\ln(3x)} = e^{-x^2\ln(3x)}\frac{d}{dx}[-x^2\ln(3x)] = -(3x)^{-x^2}[2x\ln(3x) + x].$$

1c Find $\dfrac{d}{dt}\left[\ln(\tan^{-1} t)\right]$.

$$\frac{(\tan^{-1} t)'}{\tan^{-1} t} = \frac{1}{(1+t^2)\tan^{-1} t}.$$

2a Find $\displaystyle\int \frac{6}{2x-1}\,dx$.

Let $u = 2x - 1$, so $dx = \frac{1}{2}du$ and the integral becomes

$$\int \frac{6}{2x-1}\,dx = \int \frac{3}{u}\,du = 3\ln|u| + C = 3\ln|2x-1| + C.$$

2b Find $\displaystyle\int 7^{-4x}\,dx$.

Since $7^{-4x} = e^{-4x\ln 7}$, we have

$$\int 7^{-4x}\,dx = \int e^{-4x\ln 7}\,dx = -\frac{1}{4\ln 7}e^{-4x\ln 7} + C = -\frac{7^{-4x}}{4\ln 7} + C.$$