Math 141
Summer III 2018
Exam 3

Name:

1. $\boxed{\text{10 pts. each}}$ Determine whether the series converges or diverges, using an appropriate test and justifying all work. Arguments must be clear and thorough.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} n^{-1/n}$ (d) $\displaystyle\frac{2}{3} + \frac{2\cdot 5}{3\cdot 5} + \frac{2\cdot 5\cdot 8}{3\cdot 5\cdot 7} + \frac{2\cdot 5\cdot 8\cdot 11}{3\cdot 5\cdot 7\cdot 9} + \cdots$

2. $\boxed{\text{10 pts. each}}$ Use the Alternating Series Test to show the series converges, or use some other test to show it diverges. If the series converges, use any test to determine whether it converges absolutely or conditionally.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \tan^{-1} n$

3. $\boxed{\text{10 pts.}}$ Approximate $\ln(0.9)$ with an appropriate 4th-order Taylor polynomial having an appropriate center.

4. $\boxed{\text{10 pts. each}}$ Determine the interval of convergence of the power series, making sure to test endpoints.

(a) $\displaystyle\sum \frac{3^{2n} x^n}{n^4}$ (b) $\displaystyle\sum \frac{(2x+1)^n}{n \cdot 8^n}$ (c) $\displaystyle\sum n!\,(x-10)^n$

5. $\boxed{\text{15 pts.}}$ Find a power series representation for the function

$$f(x) = \frac{5x^2}{5+x^3},$$

and determine the interval of convergence.

6. $\boxed{\text{10 pts.}}$ Use a power series to approximate the definite integral

$$\int_0^{0.3} \frac{x}{1+x^3}\,dx$$

with an absolute error less than 10^{-6}.

7. $\boxed{\text{10 pts.}}$ For the parametric equations

$$x = \sqrt{t+1}, \quad y = \sqrt{t-1}; \qquad t \in [1, \infty),$$

eliminate the parameter to obtain a Cartesian equation of the form $y = f(x)$ or $x = g(y)$. State the domain of the function.

8. $\boxed{\text{10 pts.}}$ An object moves along a straight path from the point $(4, -40)$ at time $t = 0$ to the point $(2, 10)$ at time $t = 30$. Find a parametric description of the object's path.

9. $\boxed{\text{10 pts.}}$ Convert the polar equation $r\cos\theta = \sin 2\theta$ to Cartesian coordinates, and describe the resulting curve.

10. $\boxed{\text{10 pts.}}$ Find the slope of the polar curve $r = 8\sin\theta$ at the point $(4, 5\pi/6)$.

11. $\boxed{\text{10 pts.}}$ Find the area of the region inside one loop of the polar curve $r = 2\sin 4\theta$.

Alternating Series Estimation Theorem: If $\sum(-1)^{k+1}b_k$ is a convergent alternating series such that $0 \le b_{k+1} \le b_k$ for all k, then $R_n \le b_{n+1}$ for all n.

Maclaurin Series for Some Common Functions:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \text{ for } |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \text{ for } |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \text{ for } |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}, \text{ for } -1 < x \le 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \text{ for } |x| \le 1$$

$$(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^n, \text{ for } |x| < 1, \text{ where } \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!} \text{ and } \binom{p}{0} = 1.$$

Some Trigonometric Identities:

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta, \quad \sin^2\theta = \frac{1-\cos 2\theta}{2}, \quad \cos^2\theta = \frac{1+\cos 2\theta}{2}.$$