MATH 141
SUMMER III 2018 NAME:
EXAM 2

1. [10 pts. each] Find each indefinite trigonometric integral.

(a) $\displaystyle\int \sin^3 x \cos^{-2} x \, dx$

(b) $\displaystyle\int \sec^2 x \tan^4 x \, dx$

2. [10 pts. each] Use a trigonometric substitution to evaluate the integral

(a) $\displaystyle\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}} \, dx$

(b) $\displaystyle\int \sqrt{25-36r^2} \, dr$

3. [10 pts. each] Use partial fractions to evaluate the integral

(a) $\displaystyle\int_1^2 \frac{4q^2-7q-12}{q(q+2)(q-3)} \, dq$

(b) $\displaystyle\int \frac{1}{(t^2-1)^2} \, dt$

4. [10 pts.] Evaluate the improper integral or state that it diverges:

$$\int_{-\infty}^{0} e^{bx} \, dx, \quad b > 0.$$

5. [10 pts.] Evaluate the improper integral, or state that it diverges:

$$\int_0^1 \frac{t^3}{t^4-1} \, dt.$$

6. [10 pts.] Let $\mathcal{R}$ be the region on the xy-plane bounded by

$$f(x) = \frac{1}{\sqrt{x}\ln x}$$

and the x-axis on the interval $[2, \infty)$. Find the volume of the solid generated by revolving $\mathcal{R}$ about the x-axis.

7. [10 pts. each] Find the limit of each sequence, or show that the limit does not exist.

(a) $\displaystyle\left(n \tan \frac{\pi}{n}\right)_{n=1}^{\infty}$

(b) $\left(\sqrt{n^4-2n}-n^2\right)_{n=2}^{\infty}$

8. 10 pts. Evaluate the geometric series $\displaystyle\sum_{n=2}^{\infty} \frac{3}{7^n}$.

9. 10 pts. Either show the telescoping series

$$\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$$

diverges, or find a formula for the nth partial sum s_n and evaluate $\lim_{n\to\infty} s_n$ to obtain the value of the series.

10. 10 pts. each Determine whether the series converges or diverges. Available tests: the Divergence Test and Integral Test.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^{3/2}}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$

Formulas

- $(\sin^{-1} x)' = \dfrac{1}{\sqrt{1-x^2}}, \quad (\tan^{-1} x)' = \dfrac{1}{1+x^2}, \quad (\sec^{-1} x)' = \dfrac{1}{|x|\sqrt{x^2-1}}$
- $\displaystyle\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
- $\displaystyle\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c$
- $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x\,dx$
- $\displaystyle\int \cos^n x\,dx = \frac{\cos^{n-1} x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x\,dx$
- $\displaystyle\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$
- $\displaystyle\int \sec^n x\,dx = \frac{\sec^{n-2} x\tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$
- $\displaystyle\int \tan x\,dx = \ln|\sec x| + c, \quad \int \cot x\,dx = \ln|\sin x| + c$
- $\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + c, \quad \int \csc x\,dx = -\ln|\csc x + \cot x| + c$