

1. Consider the function $f(x) = 3x + \sin x$.

(a) 5 pts. Show that f is one-to-one and hence has an inverse.

(b) 10 pts. Find $(f^{-1})'(0)$ using the Inverse Function Theorem

2. 10 pts. each Find the derivative of each function.

(a) $y = \frac{\ln(x^2)}{\ln x + 1}$

(b) $f(x) = \sin(\cos(e^x))$

(c) $g(x) = (\cot x)^{x-1}$

(d) $h(t) = t^{(t^{1/2})}$

(e) $\ell(x) = 4 \log_3(1 - x^3)$

(f) $\varphi(z) = \tan^{-1}(1/z)$

3. 10 pts. each Determine each indefinite integral.

(a) $\int \ln(3 - 2x) dx$

(b) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

(c) $\int \frac{e^{\cos x}}{\csc x} dx$

4. 10 pts. each Evaluate each definite integral.

(a) $\int_{-2}^2 \frac{e^{t/2}}{e^{t/2} + 1} dt$

(b) $\int_0^{3/2} \frac{1}{\sqrt{36 - 4x^2}} dx$

(c) $\int_0^1 \sinh^6 2y \cosh 2y dy$

5. 10 pts. Evaluate the limit using L'Hôpital's Rule: $\lim_{x \rightarrow \infty} (2x^8 - 3)^{1/\ln x}$.

6. 10 pts. Find the integral

$$\int \frac{8}{t^{-2} + 1} dt$$

7. 10 pts. each Evaluate each of the following.

(a) $\int \frac{x}{\sqrt{x+1}} dx$

(b) $\int_0^{\pi/4} x^2 \sin 2x dx$

(c) $\int (\ln x)^2 dx$

8. 10 pts. Show that if f has a continuous second derivative on $[a, b]$ and $f'(a) = f'(b) = 0$, then

$$\int_a^b x f''(x) dx = f(a) - f(b).$$

FORMULAS & DEFINITIONS

- $(\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$
- $(\tan^{-1} x)' = \frac{1}{1+x^2}$
- $(\sec^{-1} x)' = \frac{1}{|x|\sqrt{x^2-1}}$
- $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$
- $\int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c$
- $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
- $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$
- $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$
- $\int \sec^n x dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx$
- $\int \tan x dx = \ln |\sec x| + c$
- $\int \cot x dx = \ln |\sin x| + c$
- $\int \sec x dx = \ln |\sec x + \tan x| + c$
- $\int \csc x dx = -\ln |\csc x + \cot x| + c$